

Conditional Product Next Event Estimation for Gaussian Process Implicit Surface



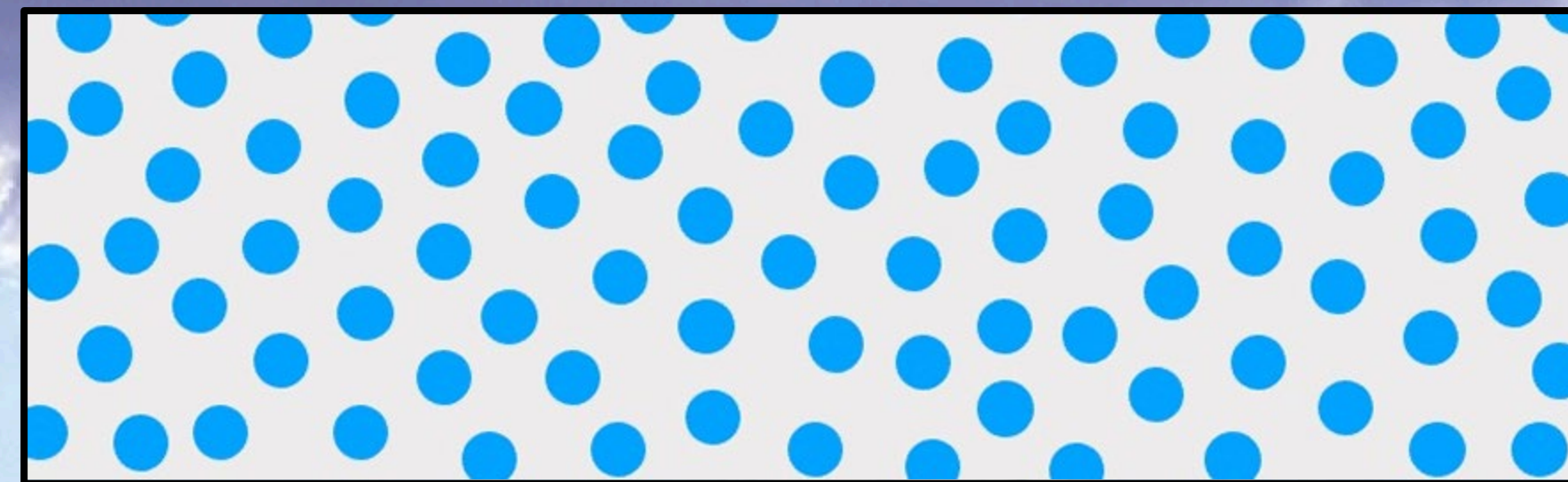
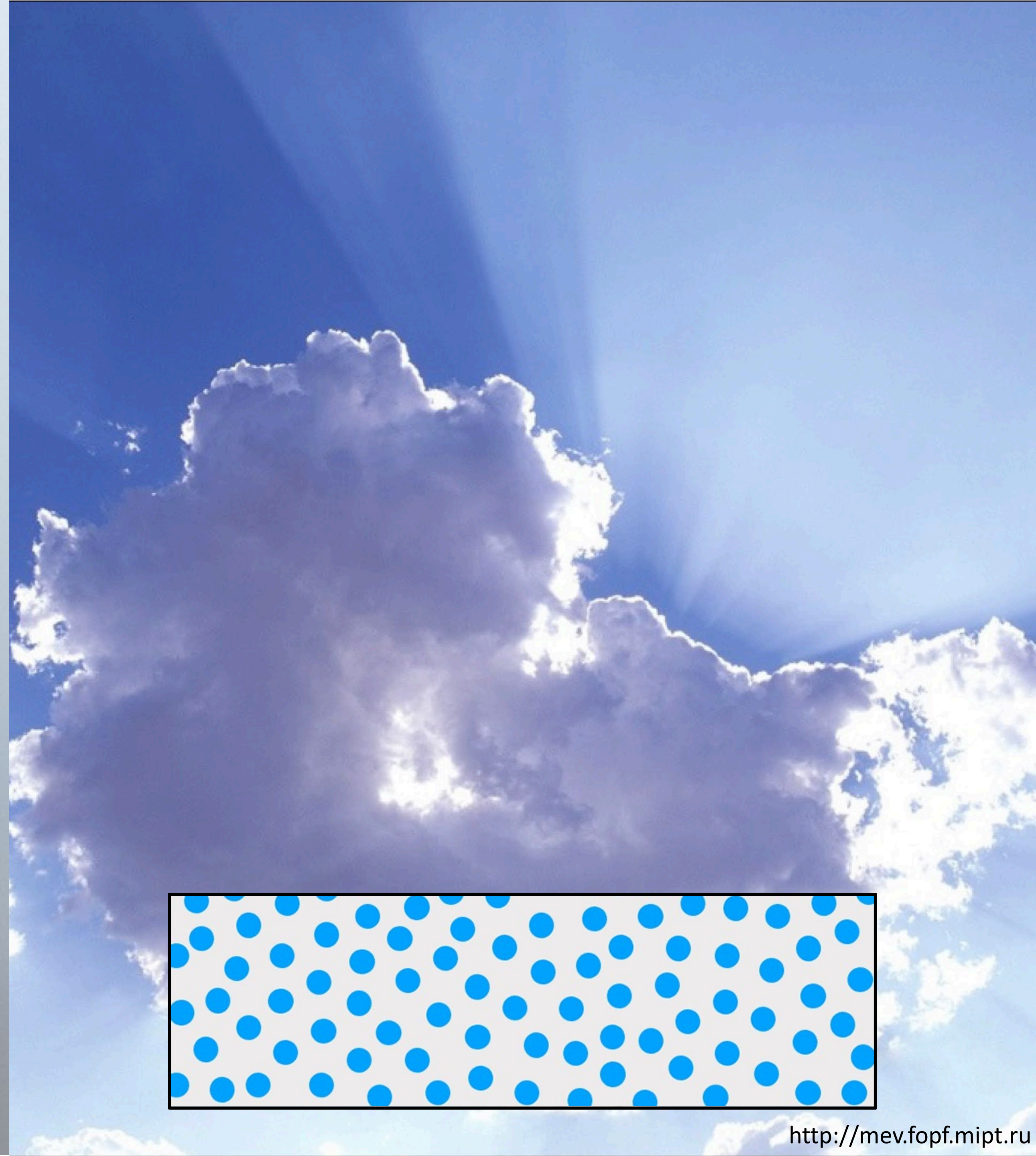
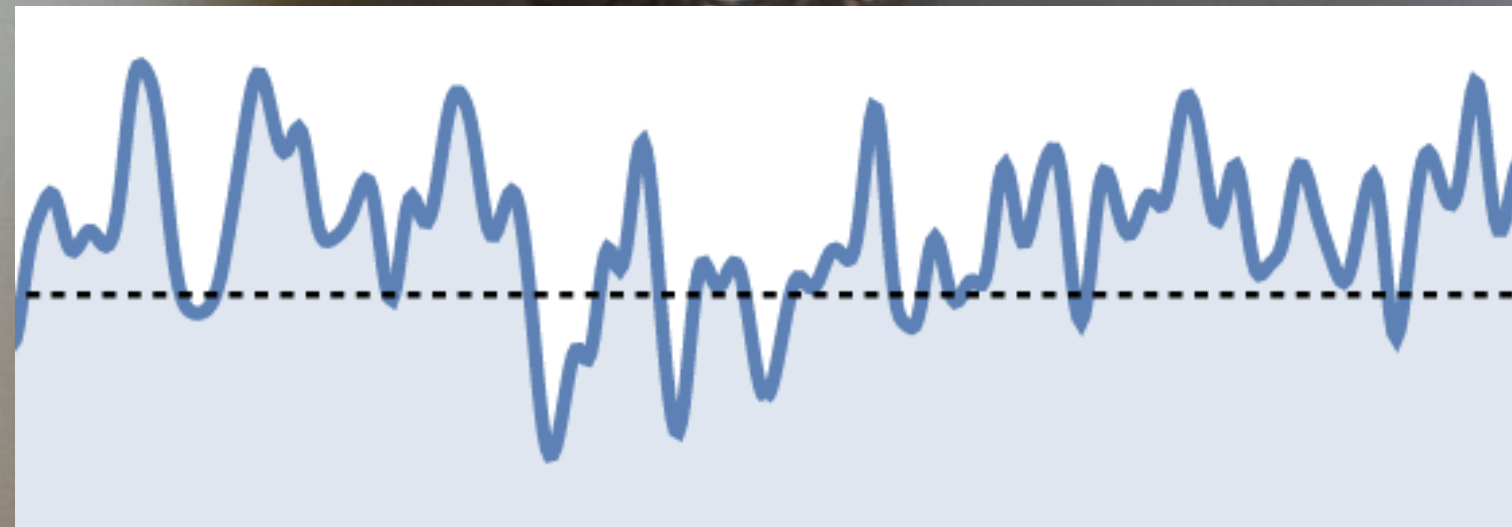
Song Shi, Kehan Xu, Wojciech Jarosz

Dartmouth College

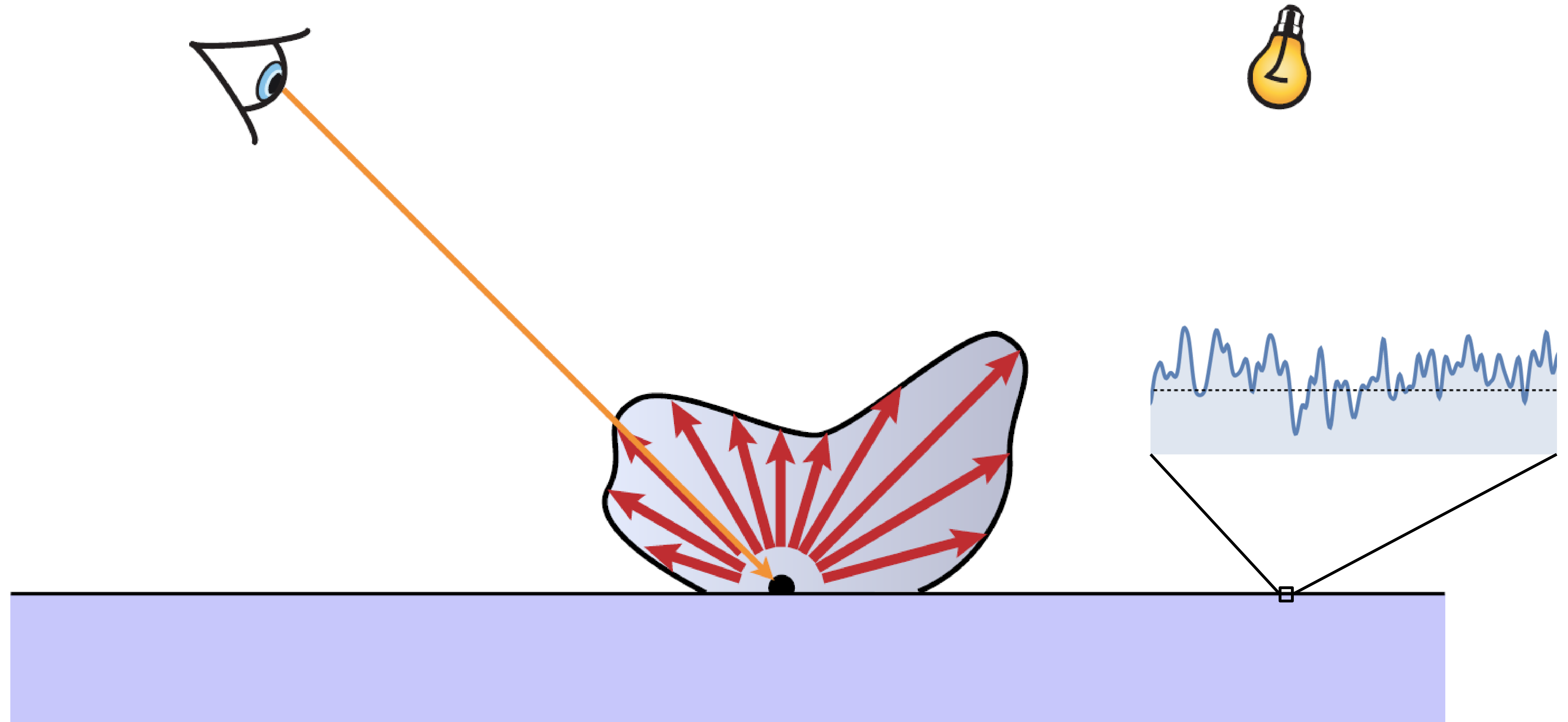
1 - 3 July 2026



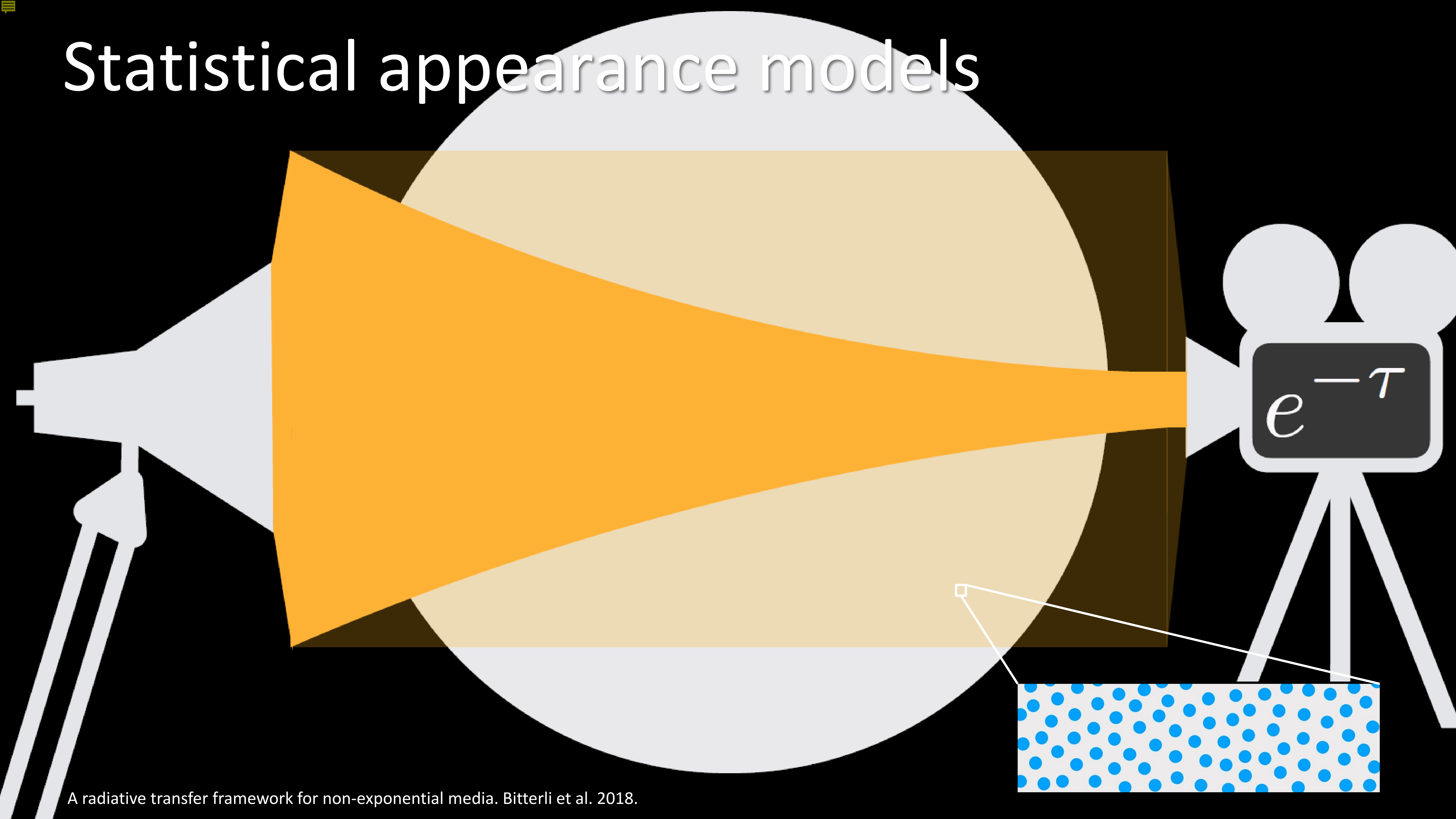
Intro



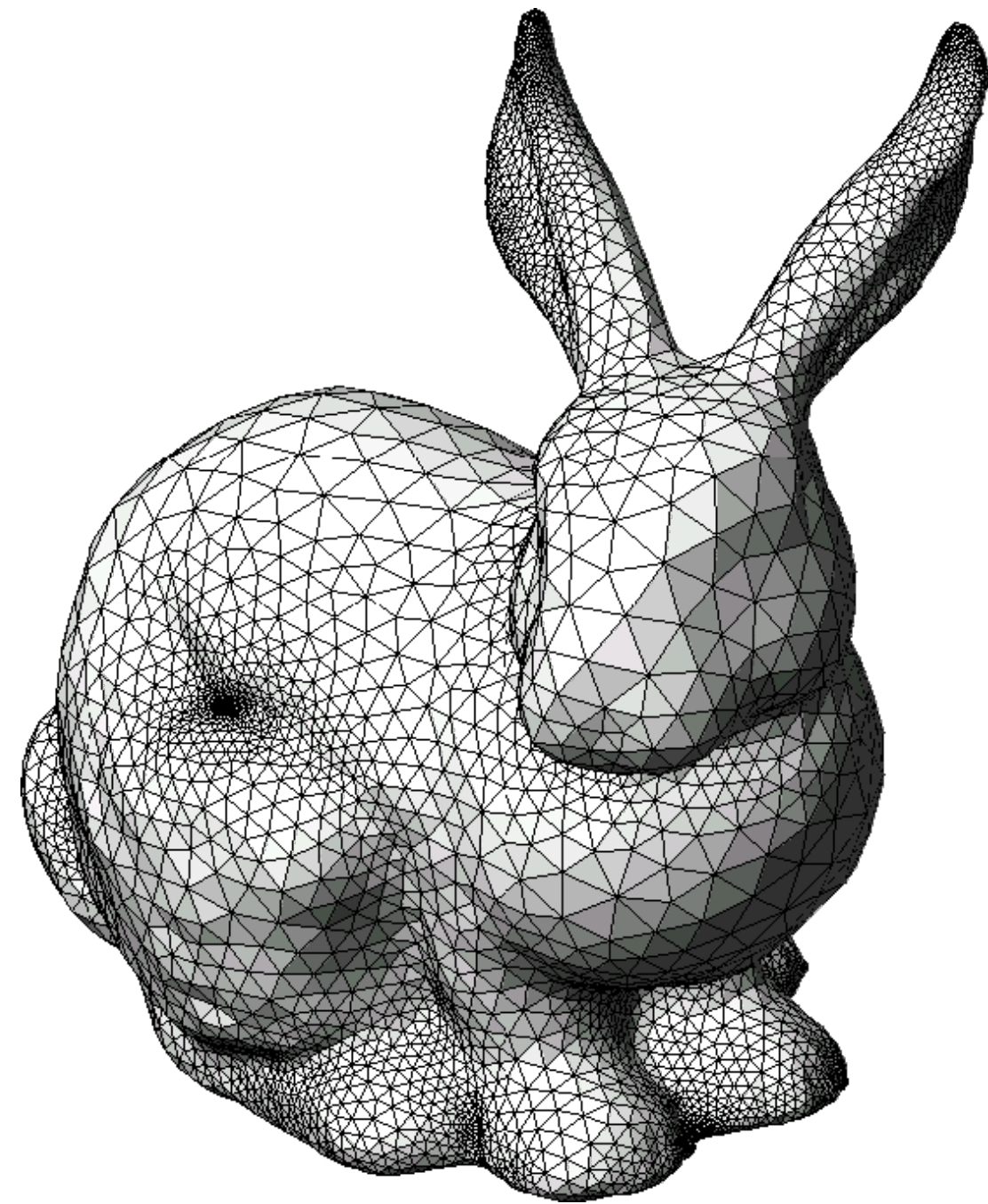
Statistical appearance models



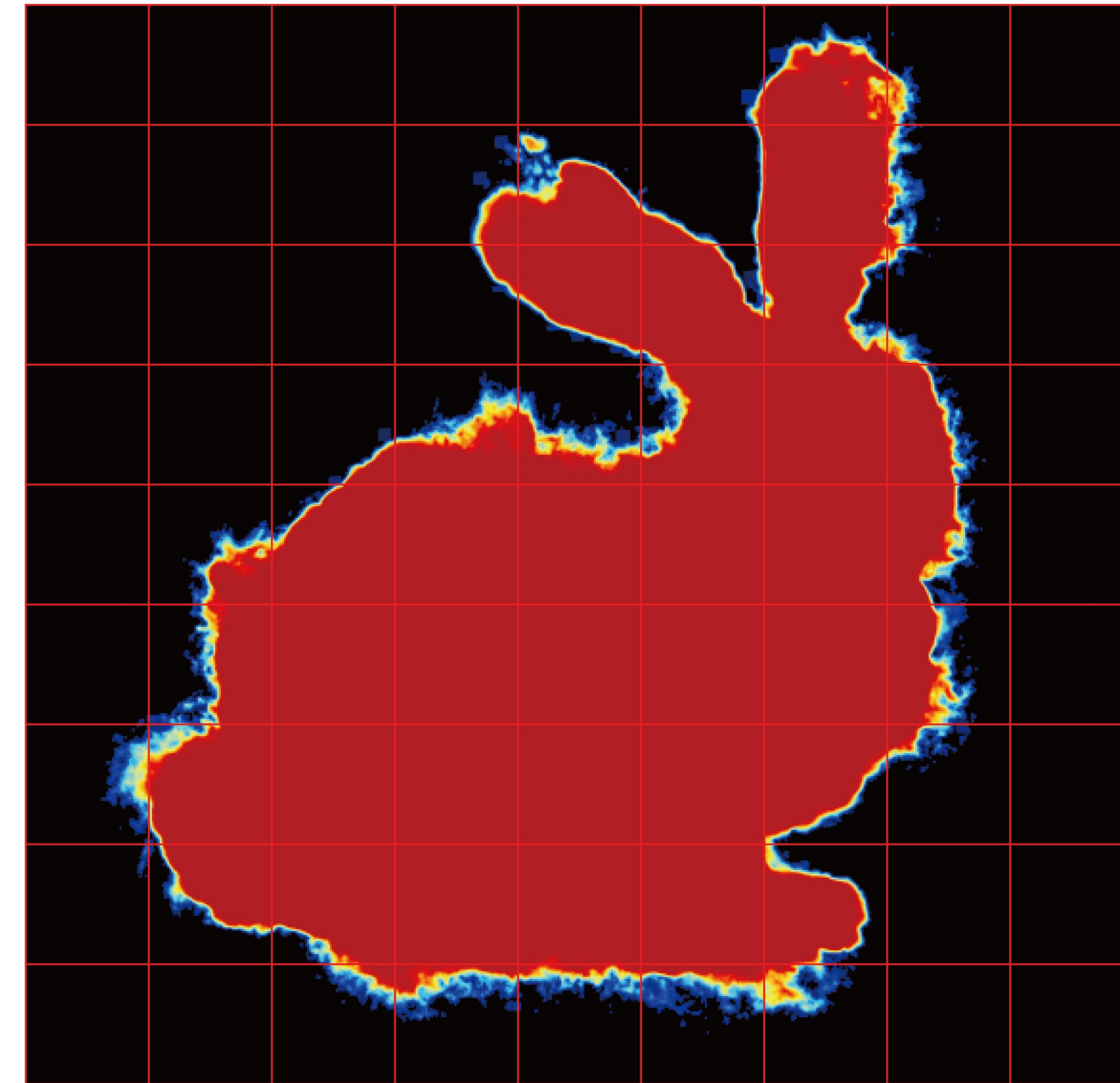
Statistical appearance models



Geometry Representations



Triangle Mesh



Voxel Density Grid

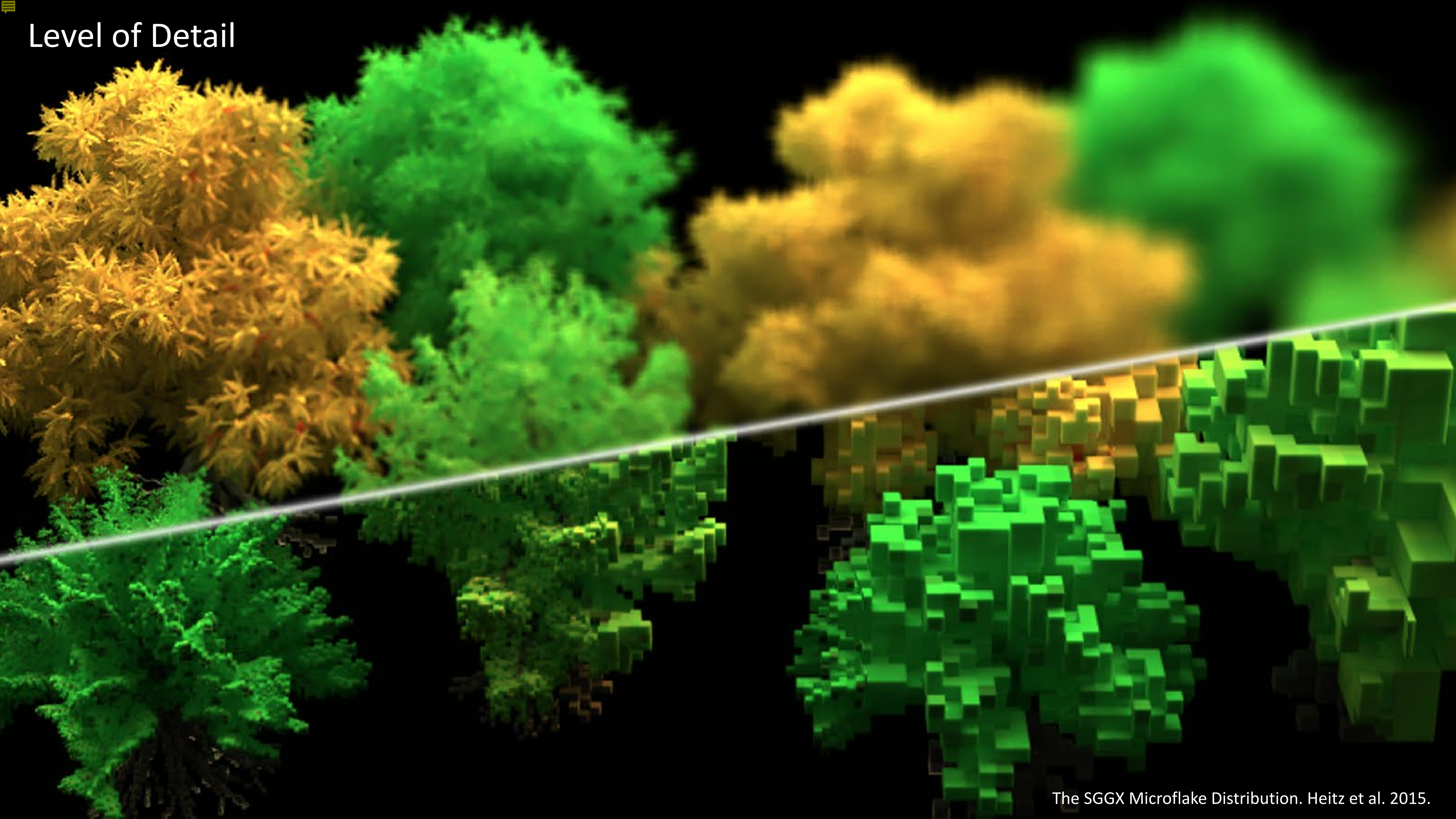
Surface or volume?





Surface or volume?

Level of Detail



Can we render
everything with
one representation
& equation?

A Unified Light Transport Theory

From microfacets to participating media: A unified theory of light transport with stochastic geometry

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BENEDIKT BITTERLI, NVIDIA, USA
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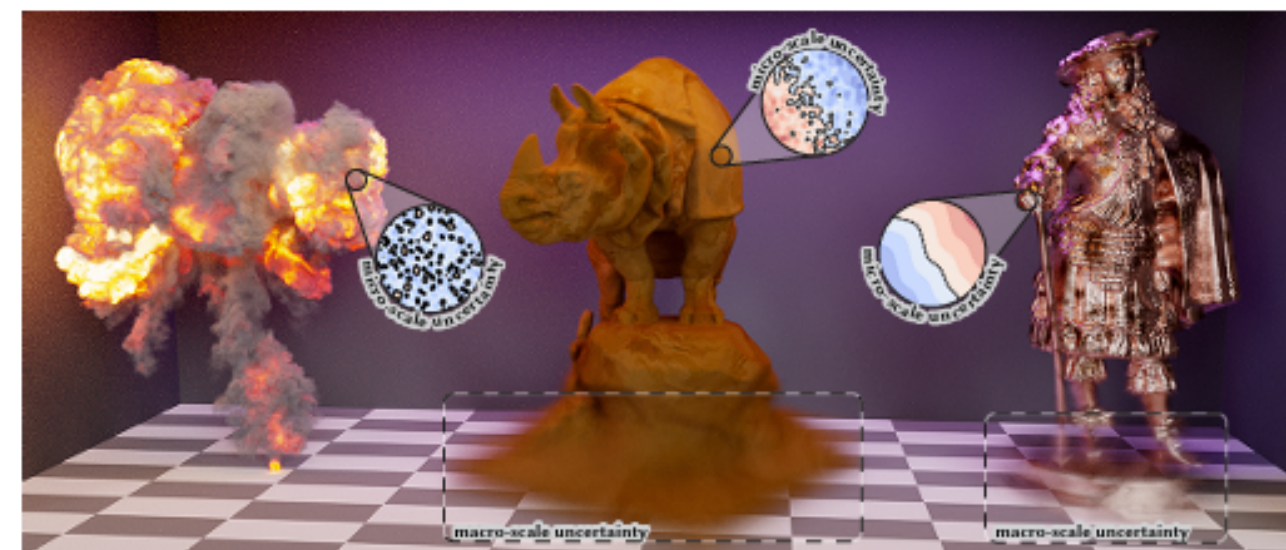


Fig. 1. We develop a theory of light transport for scenes with stochastic implicit surfaces [Dragiev et al. 2011; Sellán and Jacobson 2022; Williams and Fitzgibbon 2026] and show that this results in an expressive range of appearance behaviors that covers microfacet surfaces, classical participating media and a novel continuum in between. Each object shown above is completely described by a 3D volume that encodes the mean and covariance kernel of a non-stationary Gaussian process and is rendered using the same rendering algorithm, agnostic of its ‘appearance type’. The insets show example realizations of the process at the highlighted points in the scene. Note how the appearance of the stochastic geometry transitions from volumetric (left) to hard-surface (right) as the correlations in the process are strengthened. Increasing the variance of the process allows us to visualize uncertainty at the macro-scale (bottom).

Stochastic geometry models have enjoyed immense success in graphics for modeling interactions of light with complex phenomena such as participating media, rough surfaces, fibers, and more. Although each of these models operates on the same principle of replacing intricate geometry by a random process and deriving the average light transport across all instances thereof, they are each tailored to one specific application and are fundamentally distinct. Each type of stochastic geometry present in the scene is firmly encapsulated in its own appearance model, with its own statistics and light transport average, and no cross-talk between different models or deterministic and stochastic geometry is possible.

In this paper, we derive a theory of light transport on stochastic implicit surfaces, a geometry model capable of expressing deterministic geometry.

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microfacet surfaces, participating media, and an exciting new continuum in between containing aggregate appearance, non-classical media, and more. Our model naturally supports *spatial correlations*, missing from most existing stochastic models.

Our theory paves the way for tractable rendering of scenes in which all geometry is described by the same stochastic model, while leaving ample future work for developing efficient sampling and rendering algorithms.

CCS Concepts: • Computing methodologies → Volumetric models.

Additional Key Words and Phrases: volumetric light transport, stochastic processes, implicit surfaces

ACM Reference Format:

Dario Seyb, Eugene d'Eon, Benedikt Bitterli, and Wojciech Jarosz. 2024. From microfacets to participating media: A unified theory of light transport with stochastic geometry. *ACM Trans. Graph.* 43, 4, Article 112 (July 2024). 17 pages. <https://doi.org/10.1145/3658121>

1 INTRODUCTION

Computer graphics has seen significant progress over the past few decades, with advances in light transport, material fidelity, and

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[Seyb et al. 2024]

Practical Gaussian Process Implicit Surfaces with Sparse Convolutions

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EUGENE D'EON, NVIDIA, Switzerland
WOJCIECH JAROSZ, Dartmouth College, USA

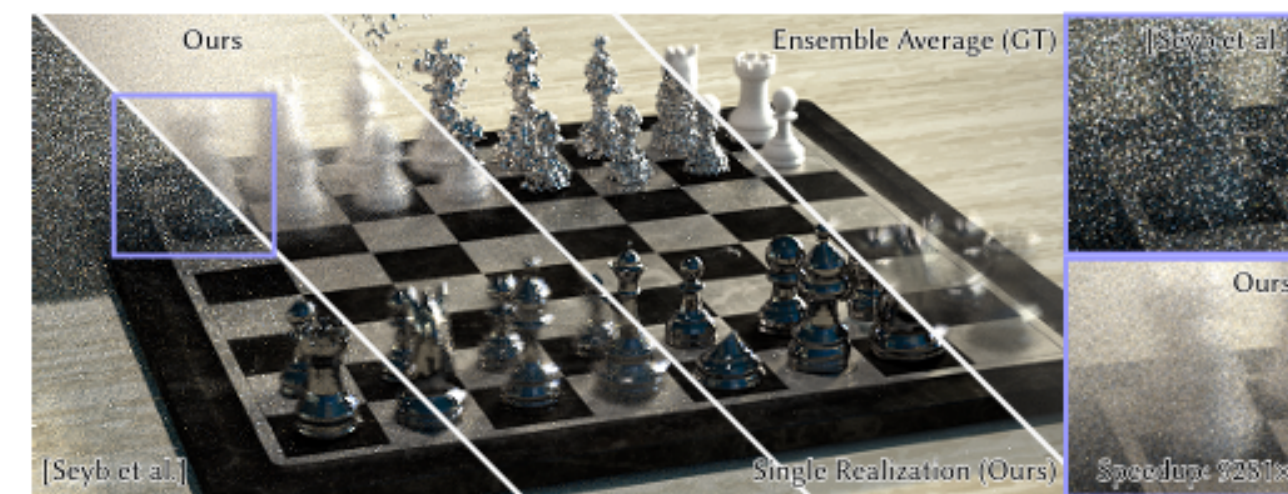


Fig. 1. Our novel sparse convolution representation for Gaussian Process Implicit Surfaces (GPISes) enables, for the first time, the visualization of single non-stationary realizations (split screen, middle right) alongside their ensemble-averaged light transport (right). The single realization allows examining the underlying structures of the stochastic geometry that lead to the aggregate rough surface or volumetric appearance. Furthermore, our algorithm (middle left) significantly boosts rendering efficiency for ensemble light transport compared to previous work by Seyb et al. [2024] (left) at equal time (40 min). The rightmost column presents magnified insets comparing the two methods, with speedup quantified by the ratio of mean squared error (MSE). This improvement stems from a combination of our efficient sparse convolution representation and the ability to perform net-error estimation even on specular micro-surfaces.

A fundamental challenge in rendering has been the dichotomy between surface and volume models. Gaussian Process Implicit Surfaces (GPISes) recently provided a unified approach for surfaces, volumes, and the spectrum in between. However, this representation remains impractical due to its high computational cost and mathematical complexity. We address these limitations by reformulating GPISes as procedural noise, eliminating expensive linear system solves while maintaining control over spatial correlations. Our method enables efficient sampling of stochastic realizations and supports flexible conditioning of values and derivatives through pathwise updates. To further enable practical rendering, we derive analytic distributions for surface normals, allowing for variance-reduced light transport via next-event estimation and multiple importance sampling. Our framework achieves efficient, high-quality rendering of stochastic surfaces and volumes with significantly simplified implementations on both CPU and GPU, while preserving the generality of the original GPIS representation.

CCS Concepts: • Computing methodologies → Rendering; Ray tracing.

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Graphics*, <https://doi.org/10.1145/3768329>.

Additional Key Words and Phrases: stochastic processes, implicit surfaces

ACM Reference Format:

Kehan Xu, Benedikt Bitterli, Eugene d'Eon, and Wojciech Jarosz. 2025. Practical Gaussian Process Implicit Surfaces with Sparse Convolutions. *ACM Trans. Graph.* 44, 6, Article 188 (December 2025), 18 pages. <https://doi.org/10.1145/3763329>

1 INTRODUCTION

Traditional light transport simulation categorizes scene components as either solid surfaces or participating media, with each governed by a distinct rendering equation. This binary distinction is inadequate for capturing the diverse appearances found in the real world, particularly when considering level-of-detail techniques that might require turning hard surfaces into volumetric effects with surface-like correlation.

Seyb et al. [2024] demonstrated a unified framework using ensemble-averaged light transport through stochastic geometry, covering surfaces, volumes, and the spectrum in between. A key contribution was establishing a rigorous mathematical formalism for ensemble-averaged transport. Despite its potential, this method remains impractical, with rendering speeds at least two orders of magnitude slower than conventional path tracing. This performance gap arises

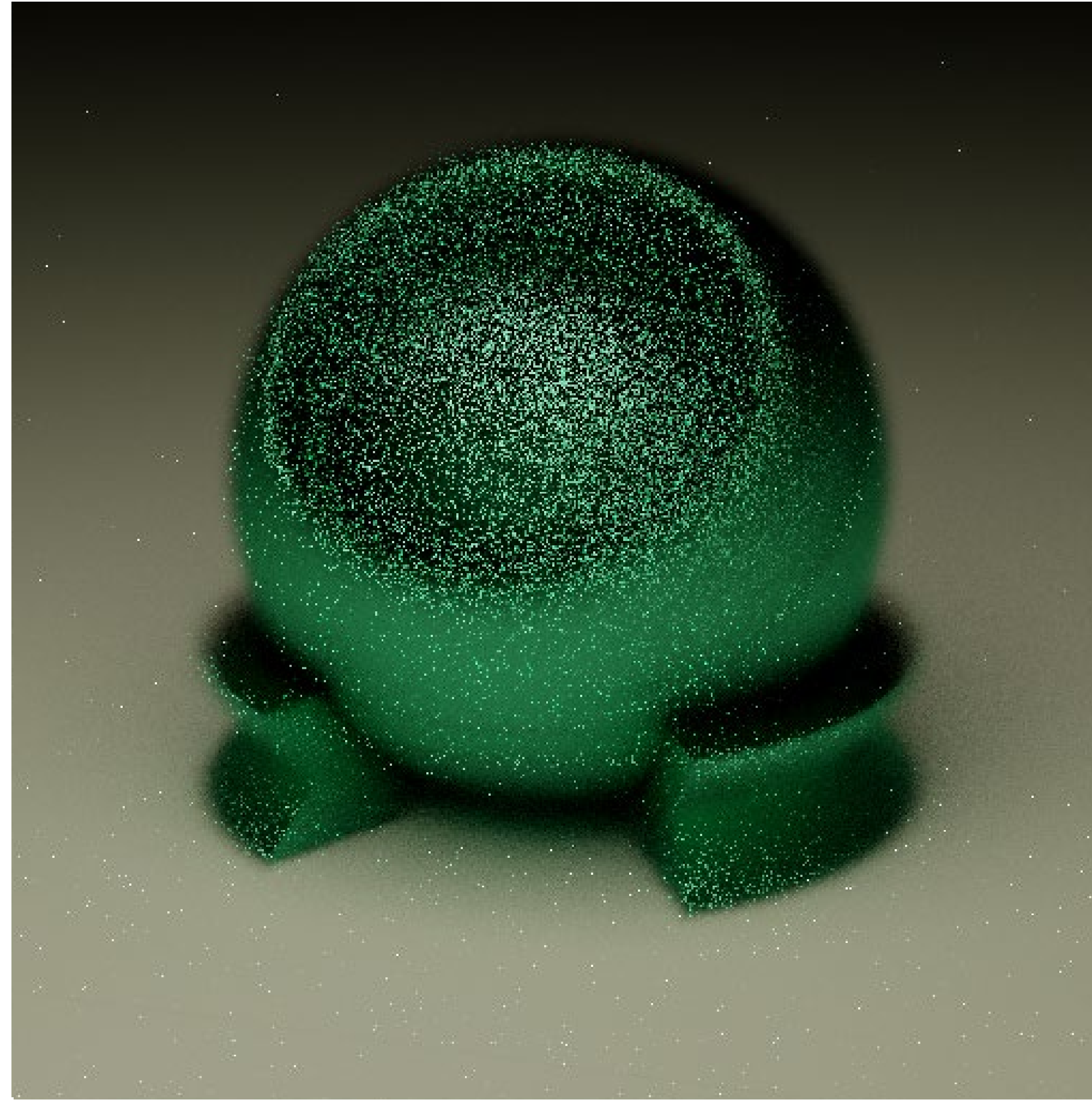
ACM Trans. Graph., Vol. 44, No. 6, Article 188. Publication date: December 2025.

[Xu et al. 2025]



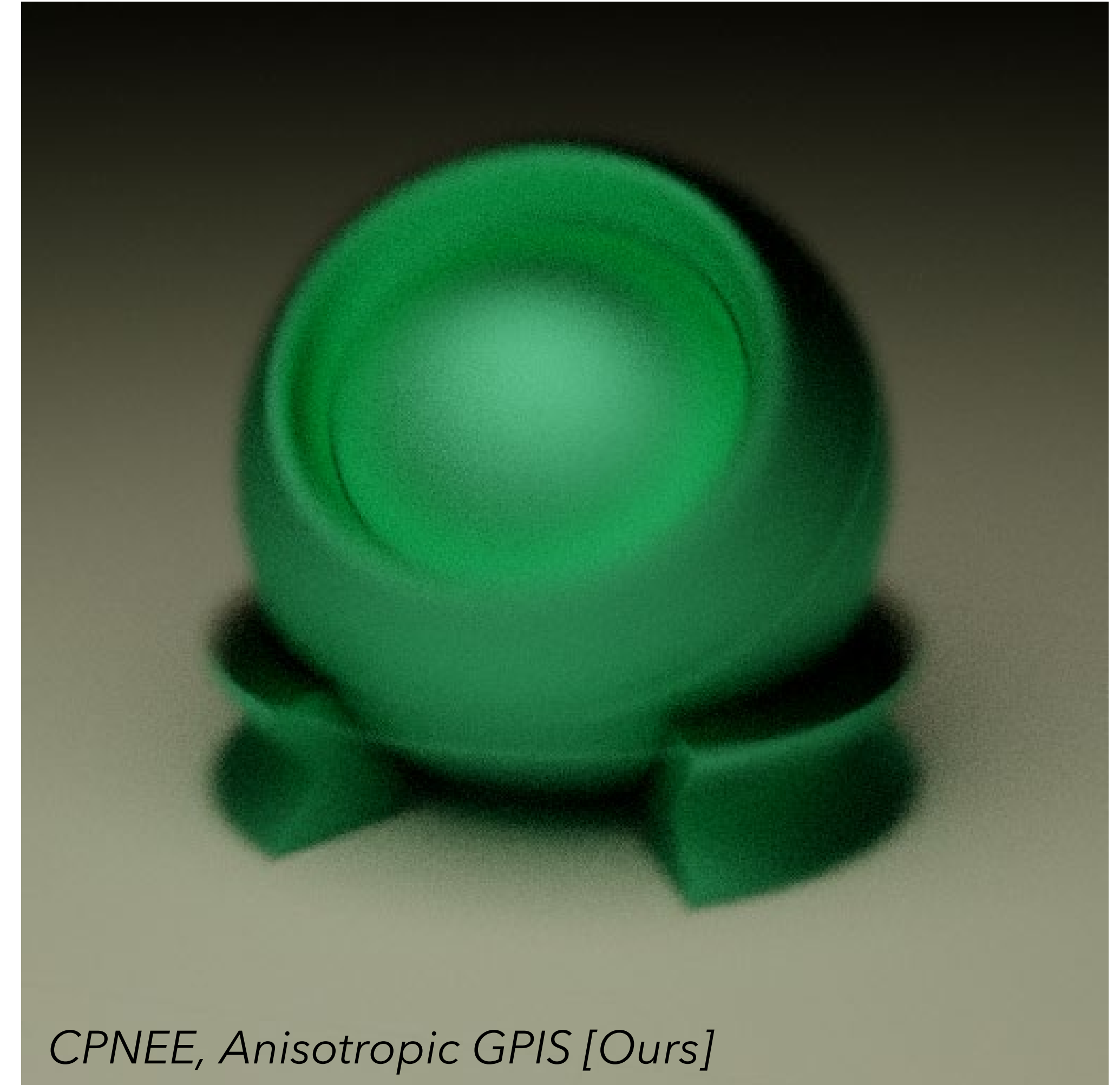
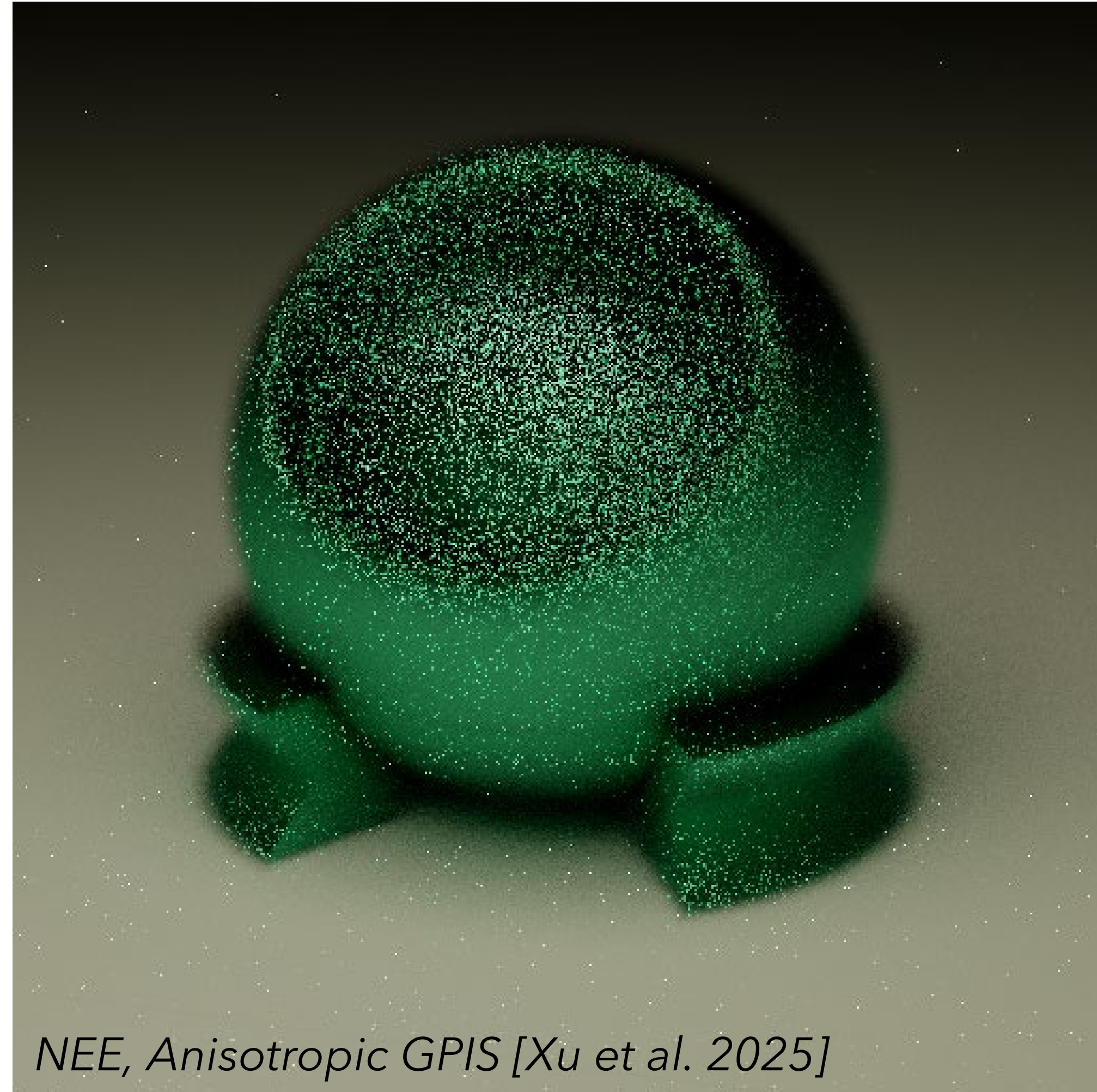
Problem Statement

Problem Statement



Volume Surface

Problem Statement

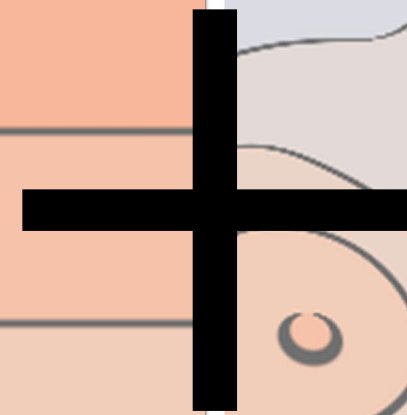




Background

Gaussian Process

Mean Function

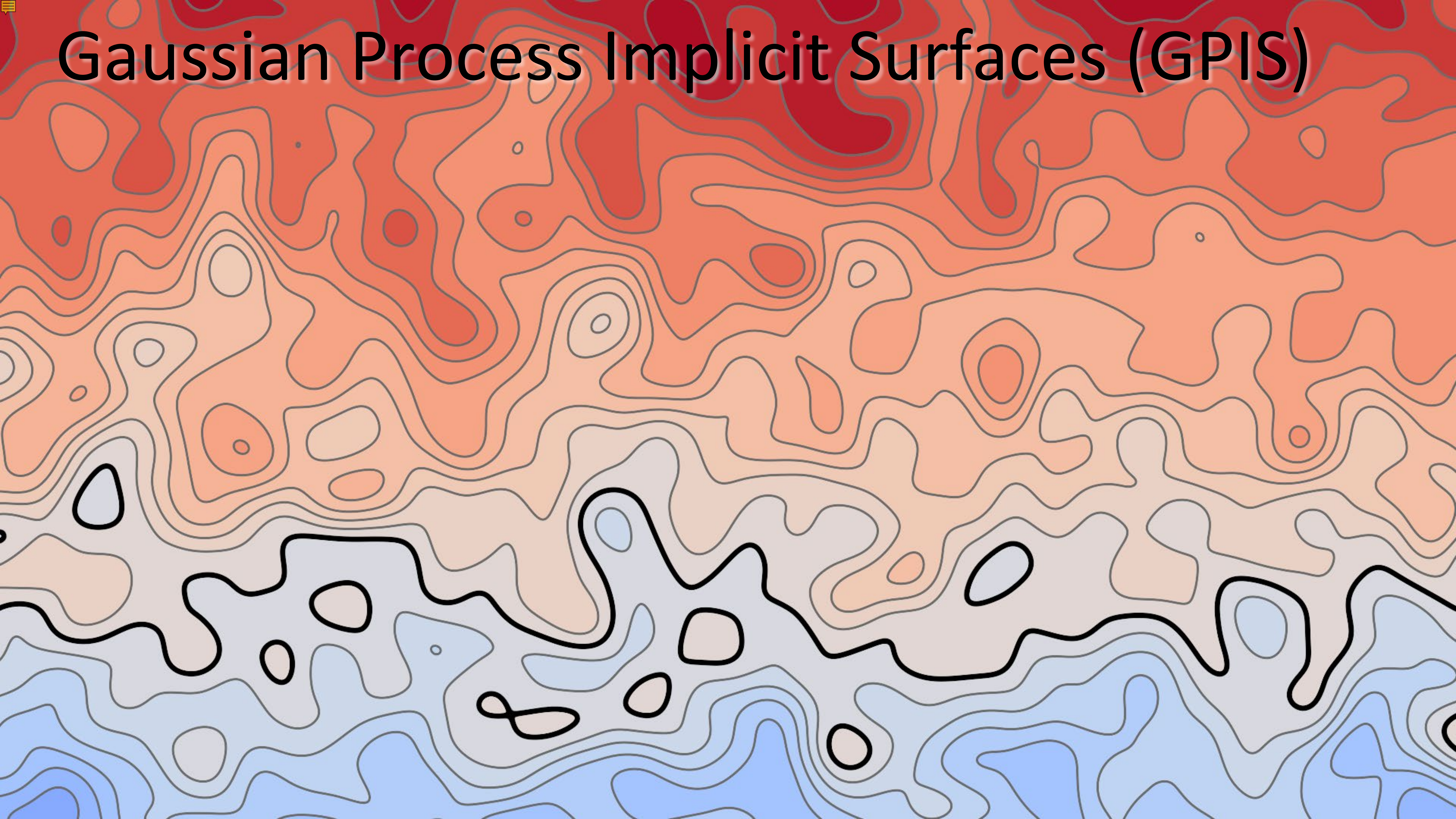


Noise Function

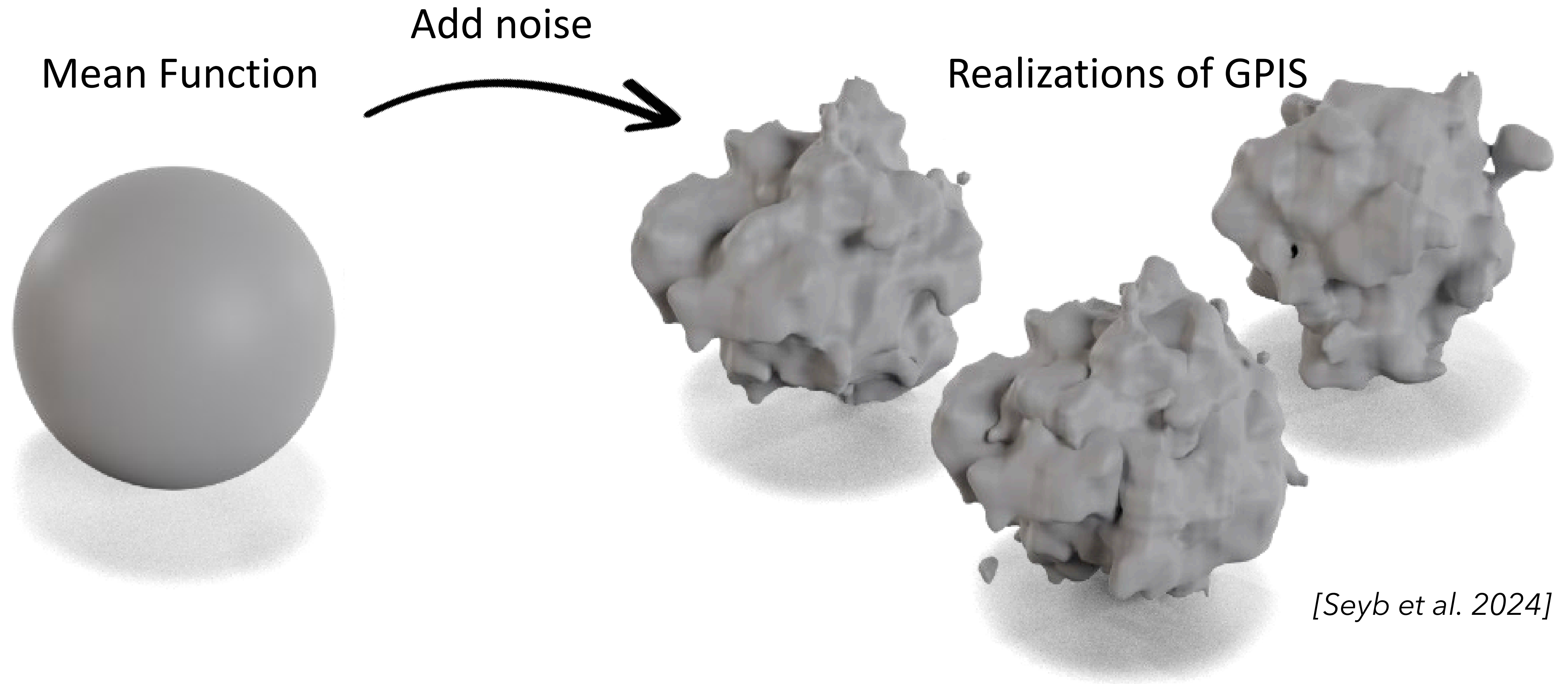


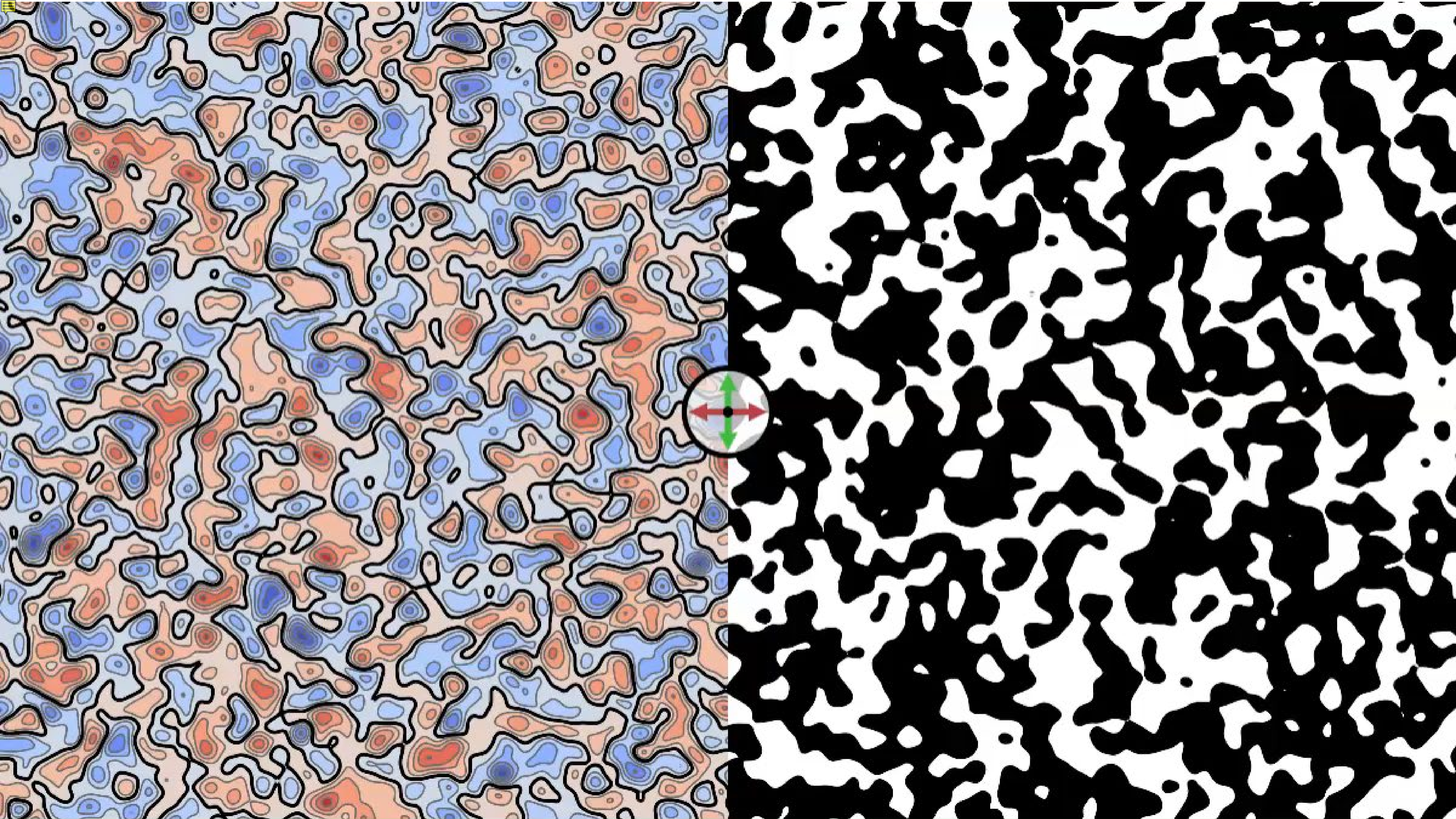
Gaussian Process

Gaussian Process Implicit Surfaces (GPIS)

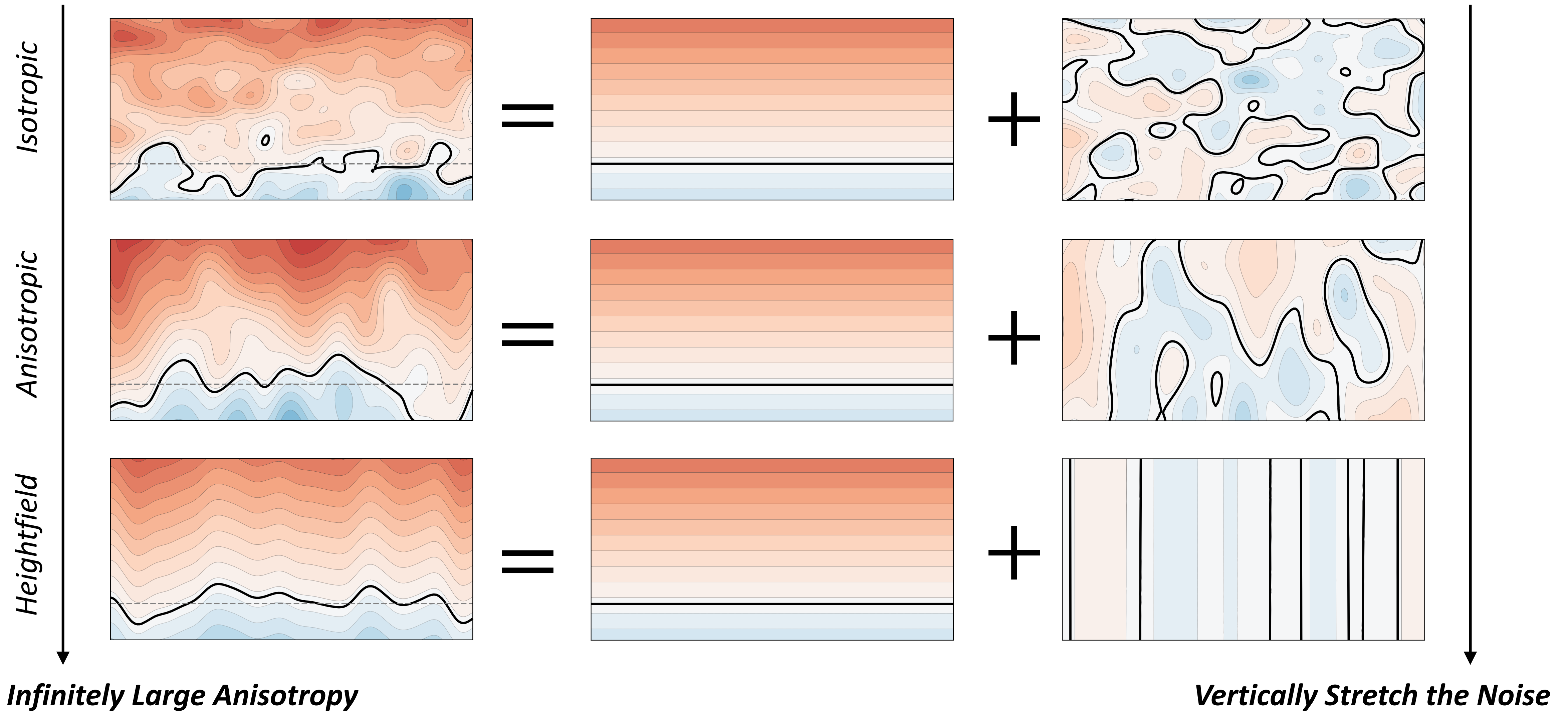


(Realizations of) Gaussian Process Implicit Surfaces





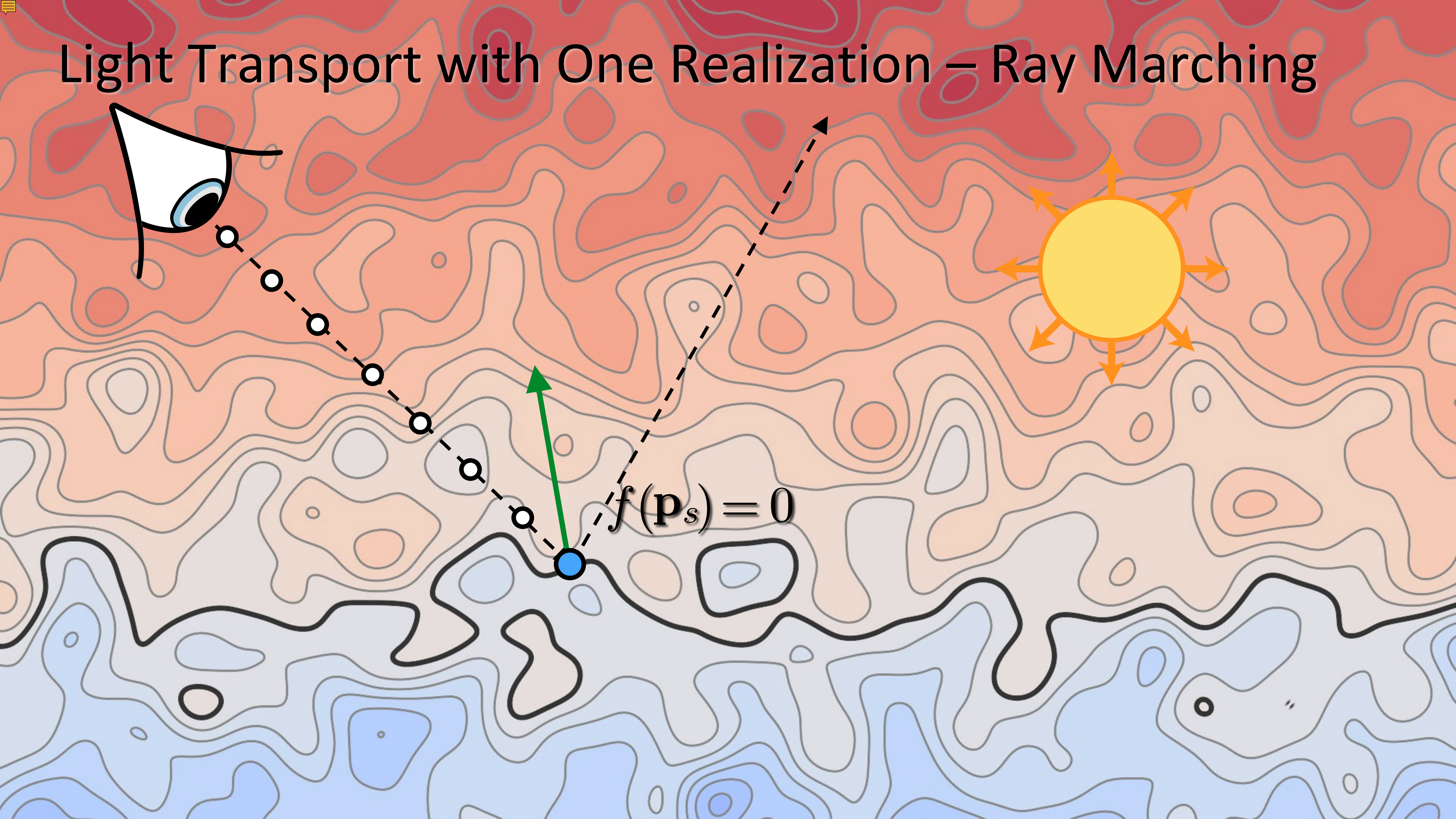
Gaussian Process Implicit Surfaces





Previous Methods

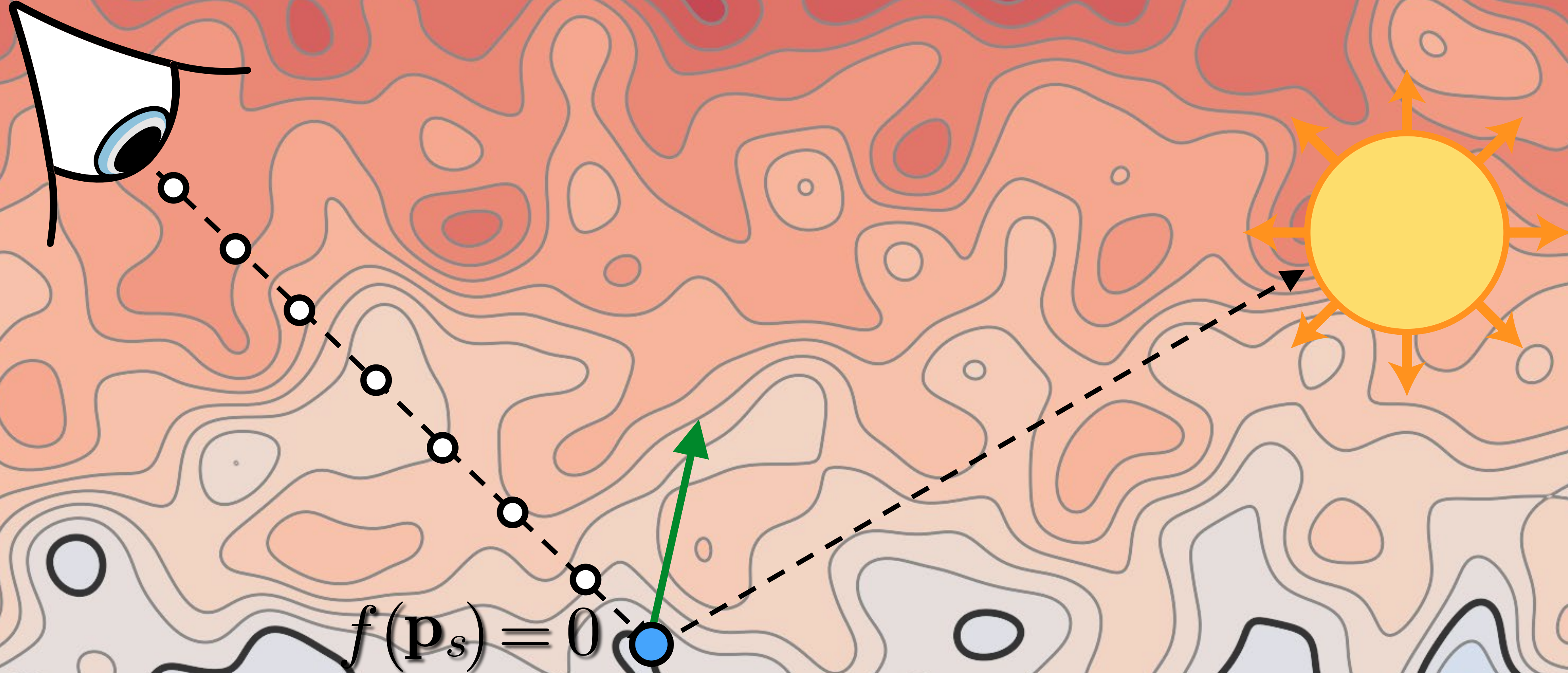
Light Transport with One Realization – Ray Marching



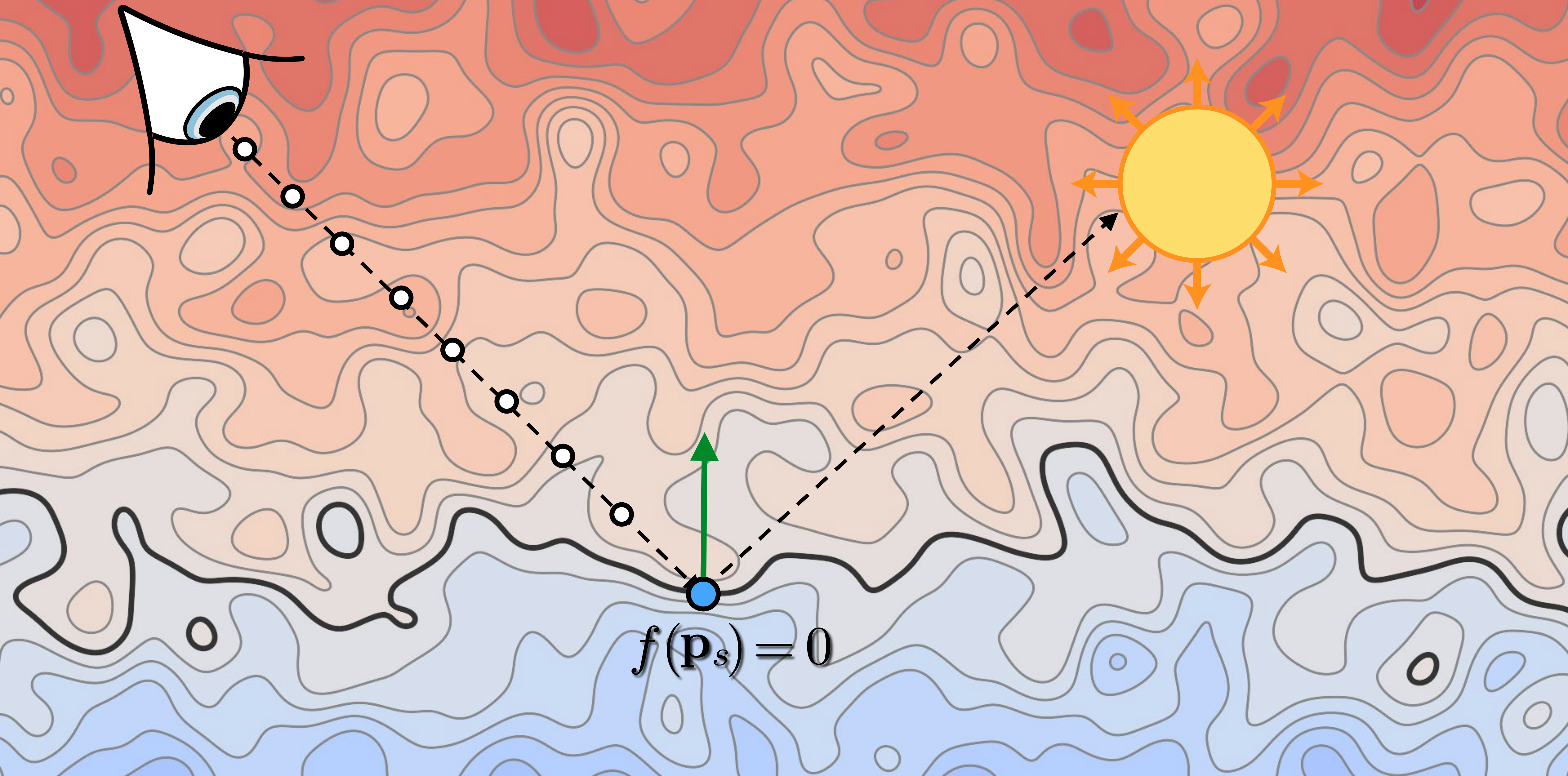
Light Transport with One Realization – Ray Marching



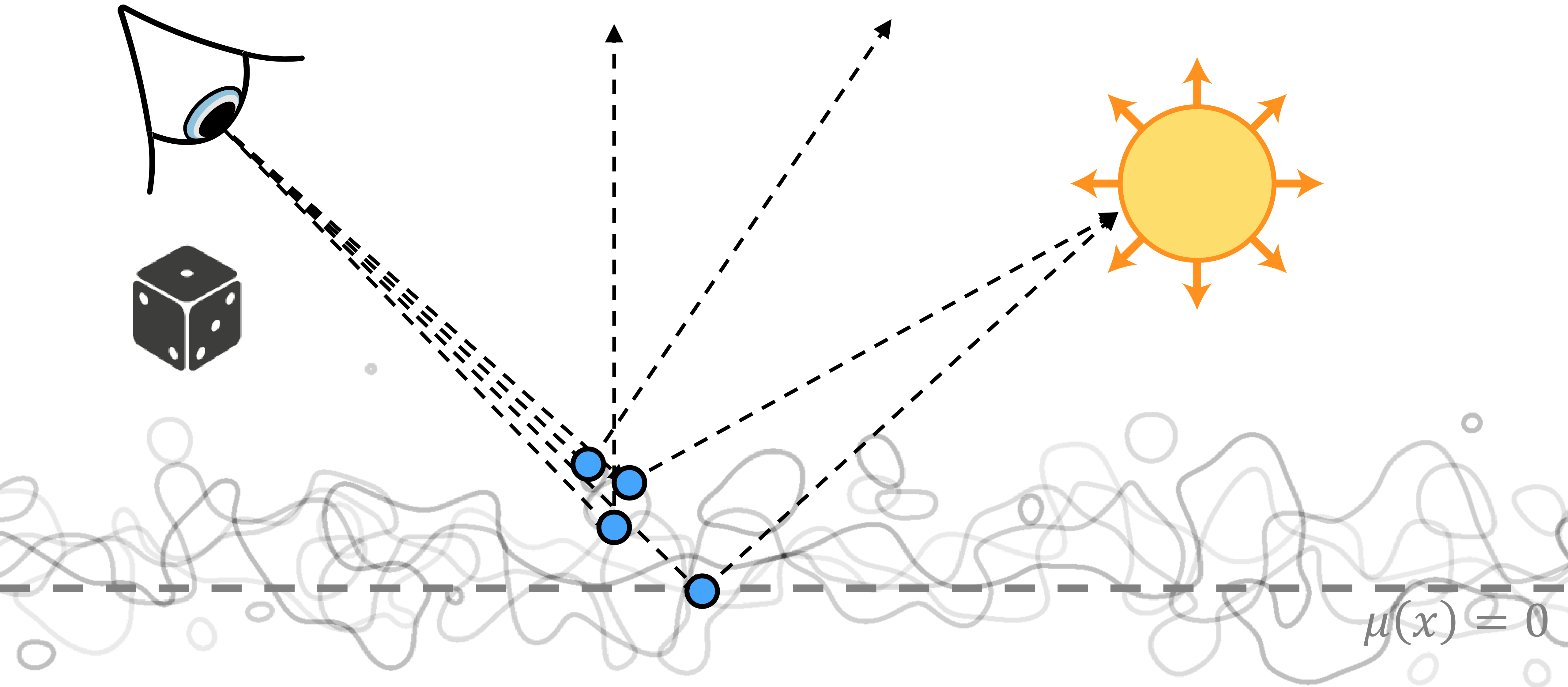
Light Transport with Another Realization



Light Transport with Another Realization



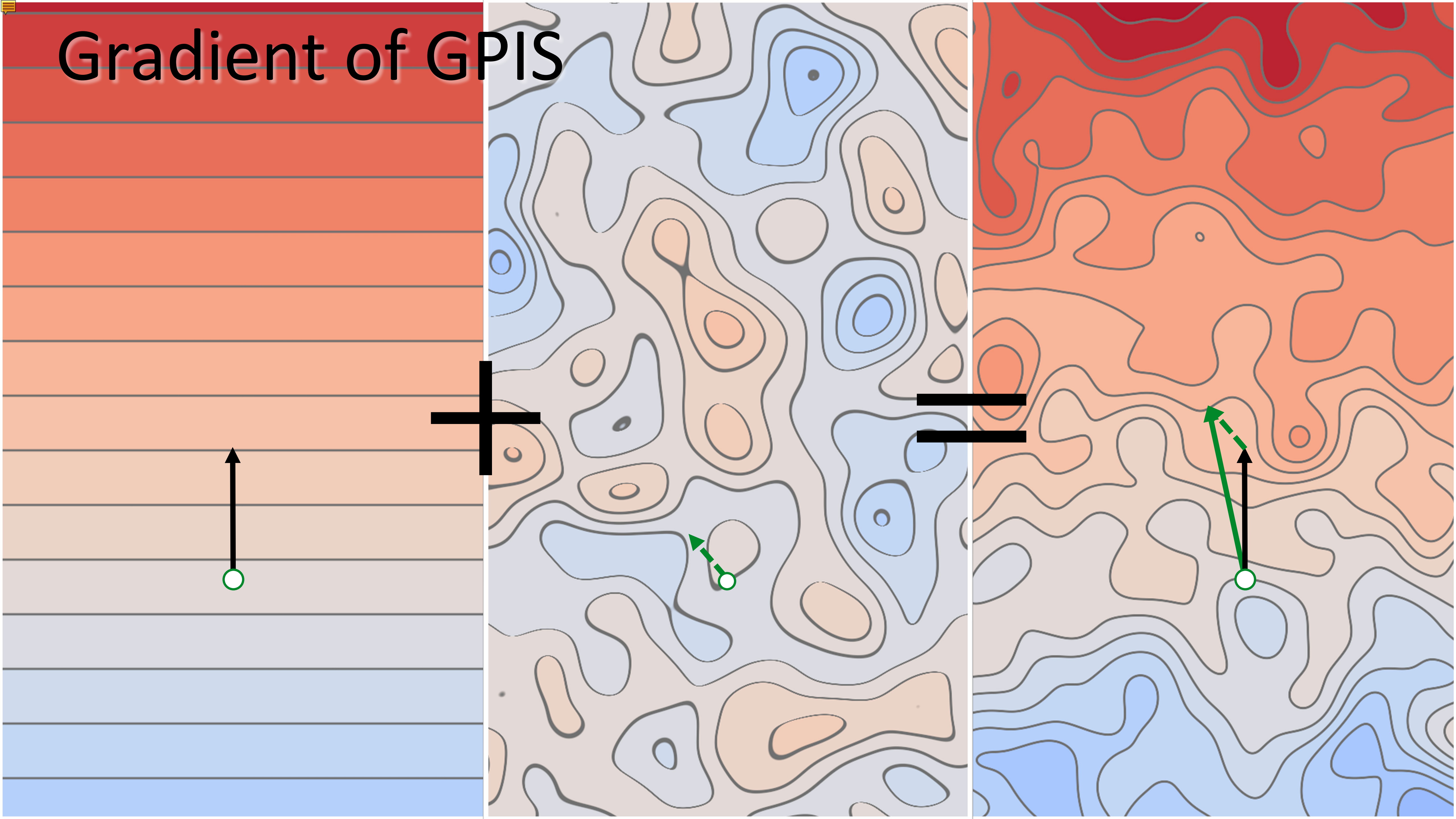
Ensemble-Averaged Light Transport



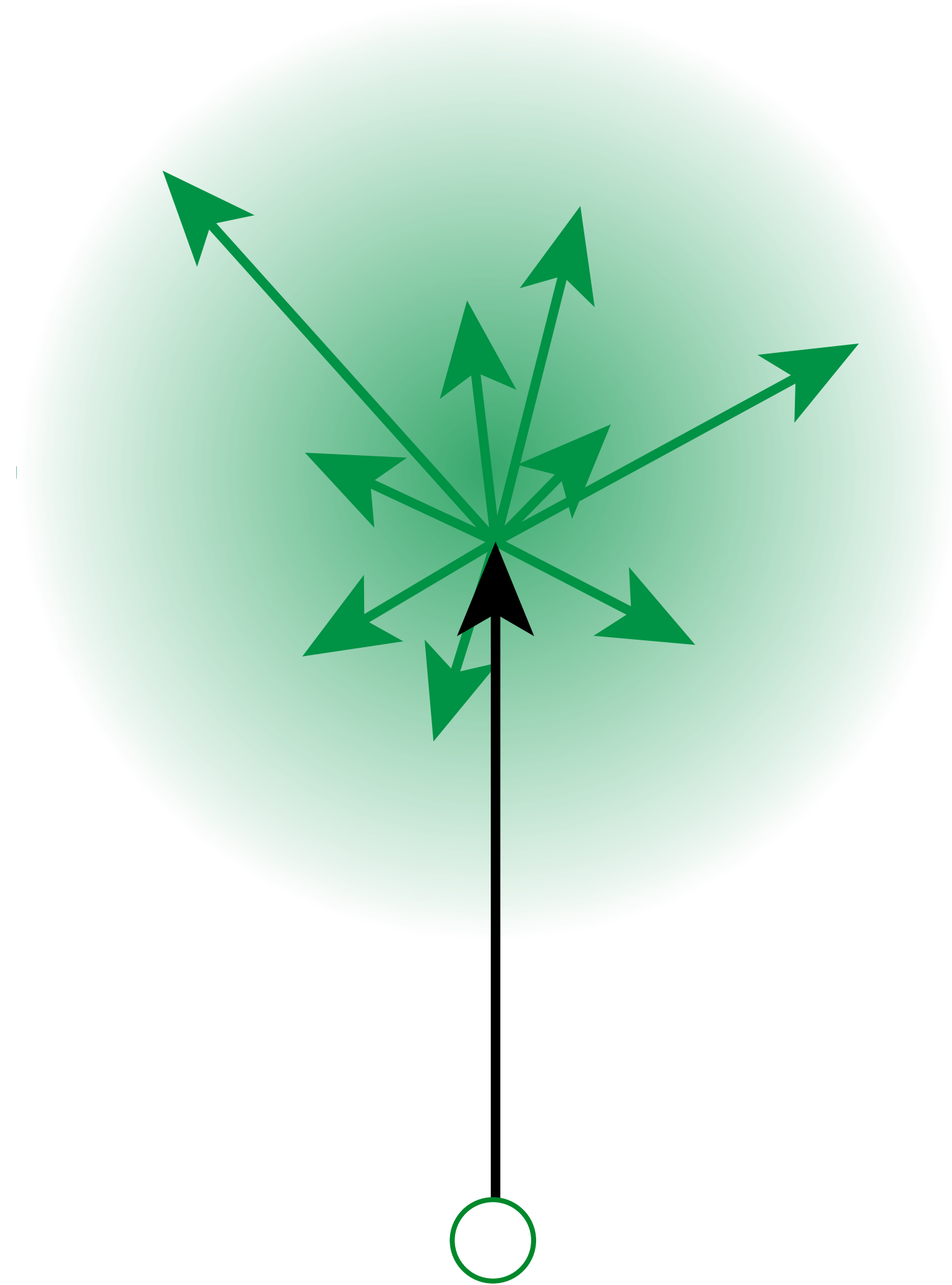
Ensemble-Averaged Light Transport



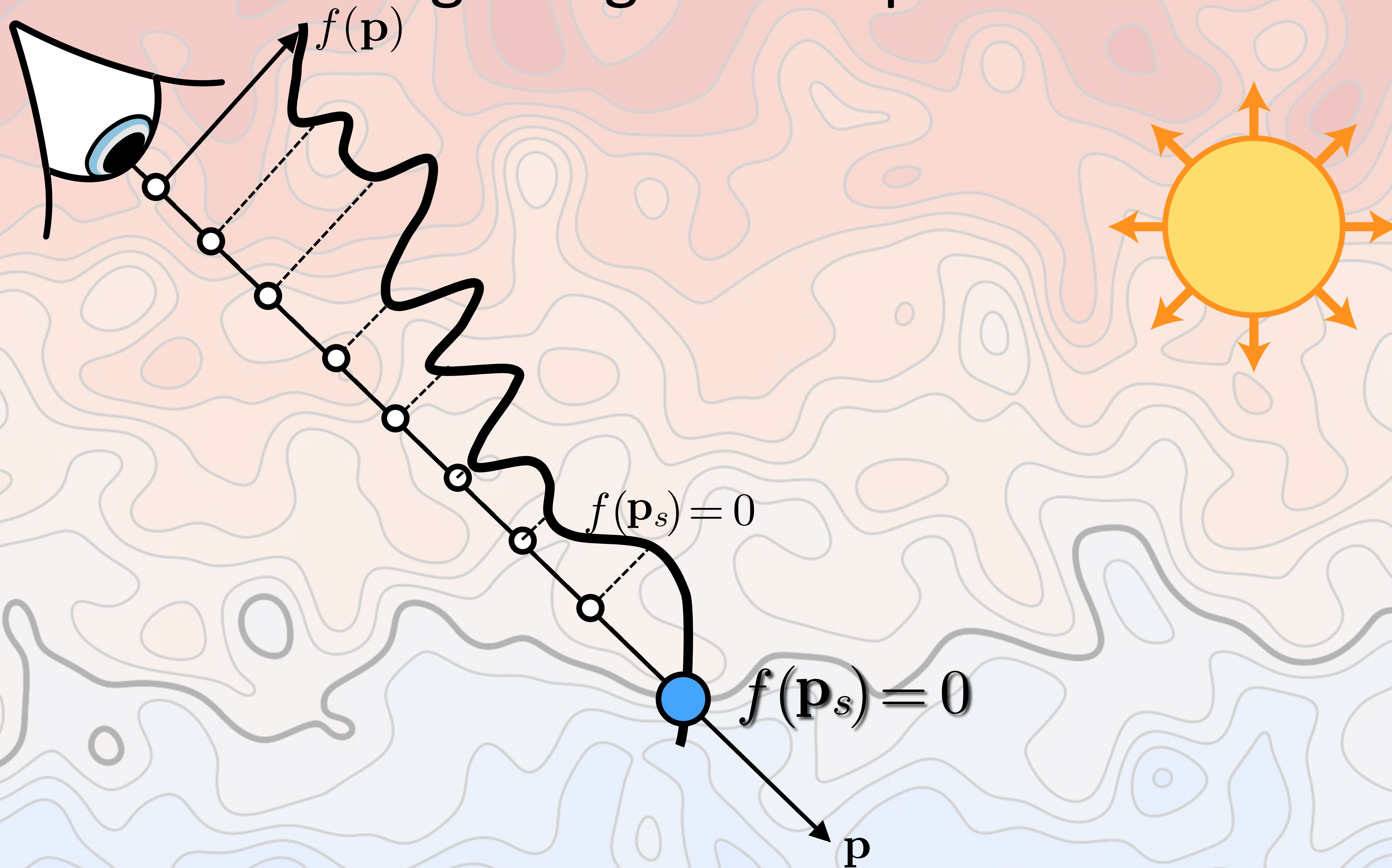
Gradient of GPIS



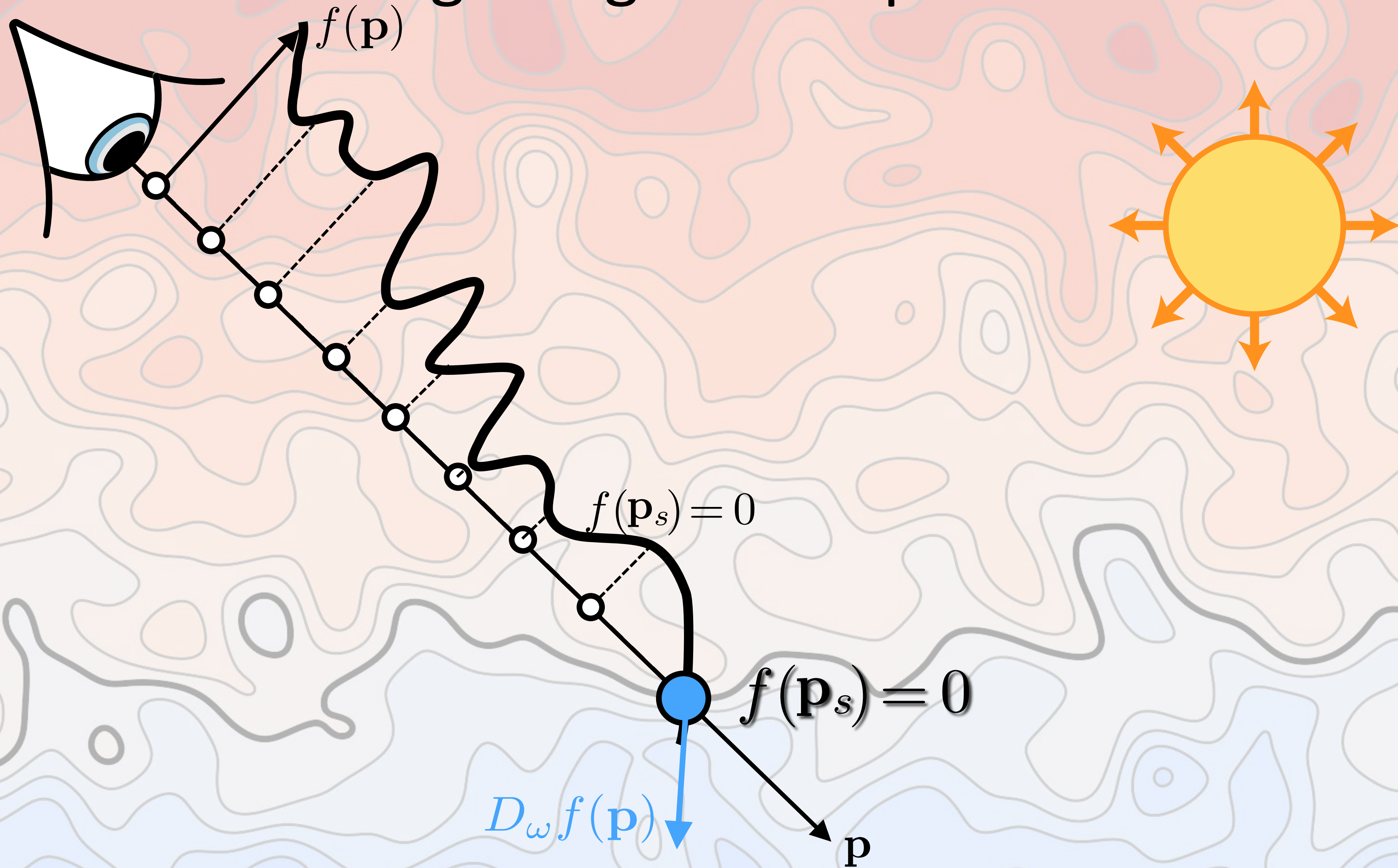
Distribution of Gradient



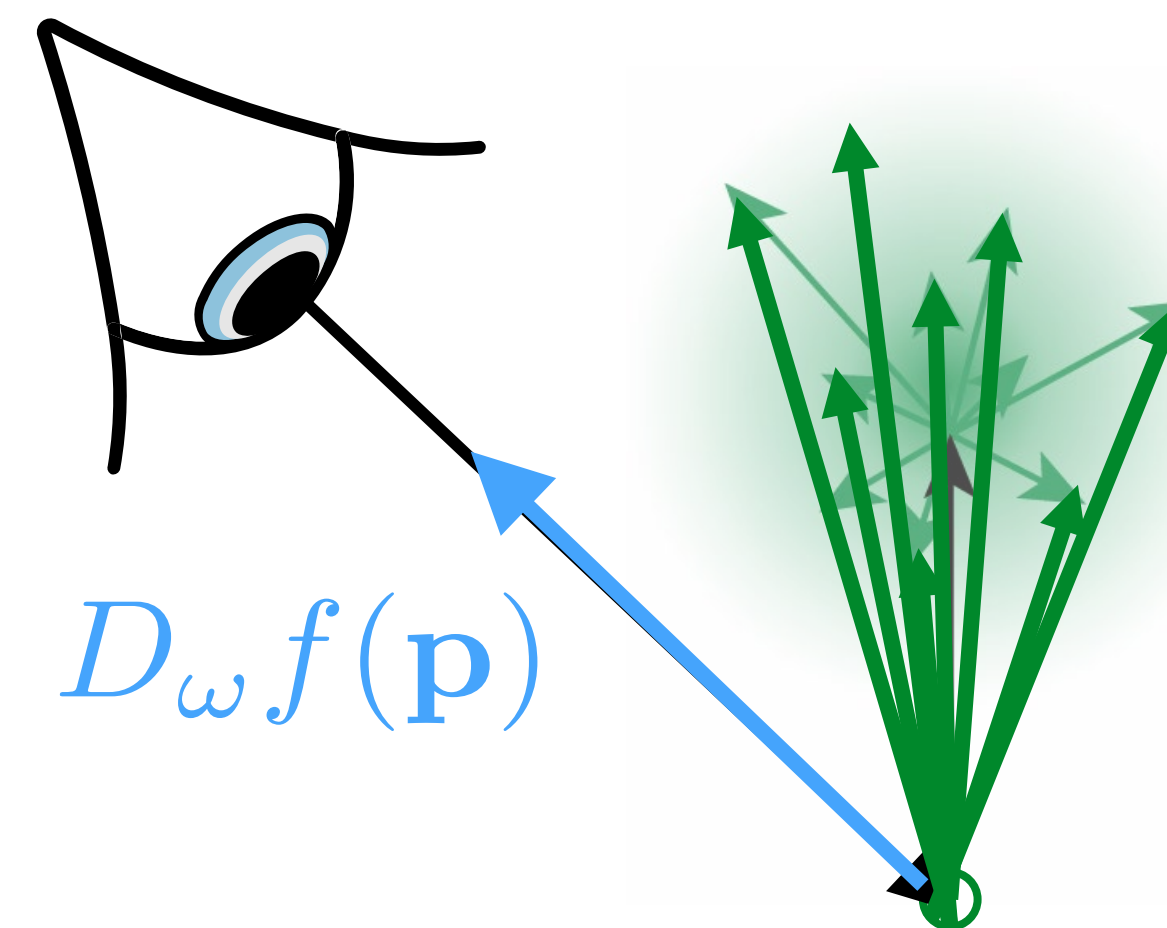
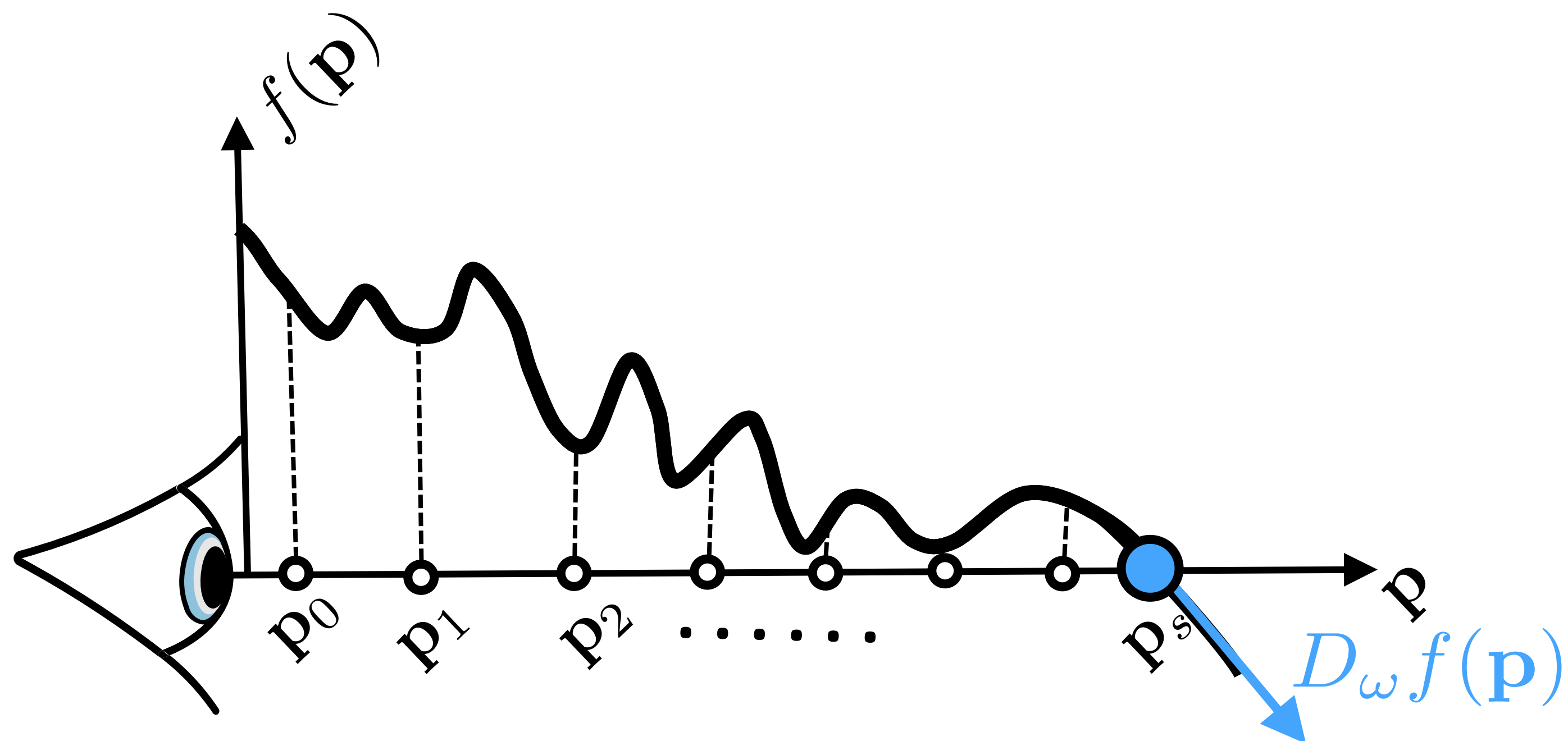
Ensemble-Averaged Light Transport



Ensemble-Averaged Light Transport



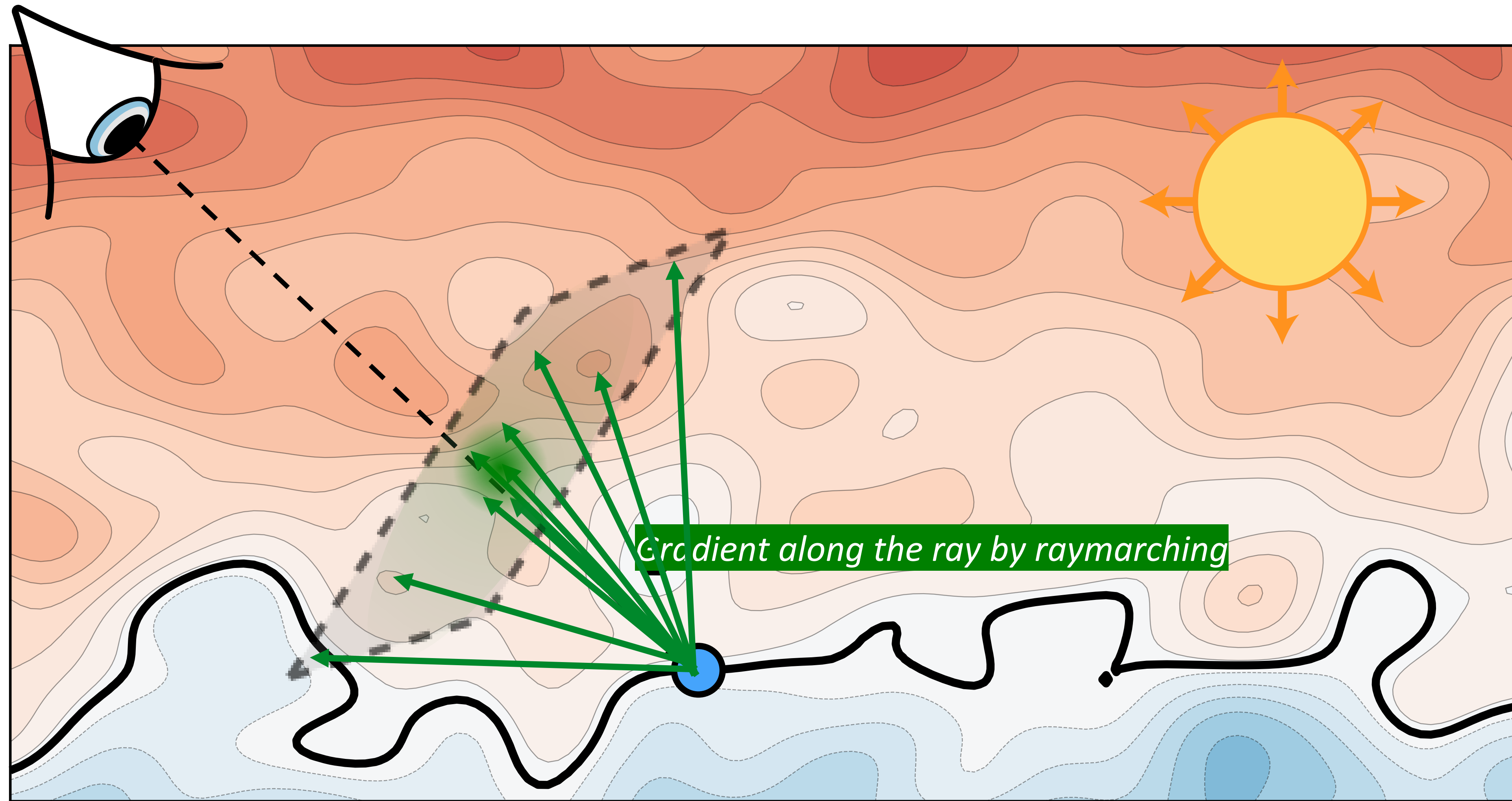
Ensemble-Averaged Light Transport – Directional Derivative



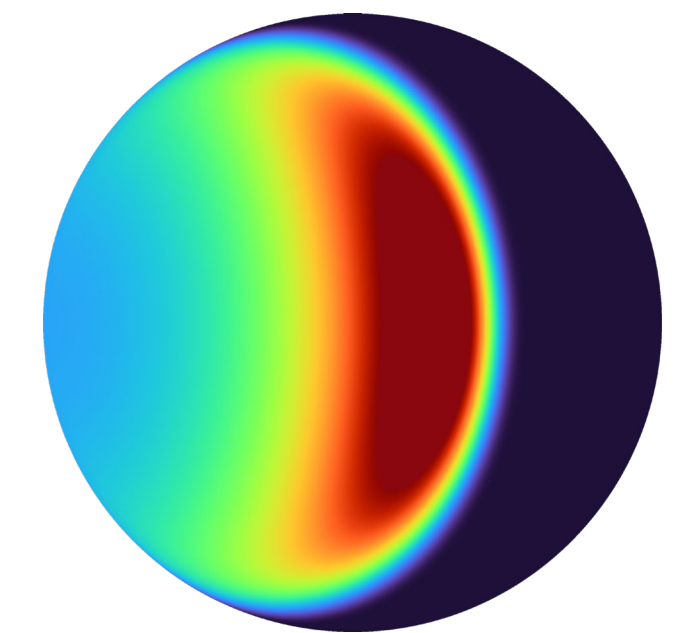
$$\begin{matrix} g_u \\ g_v \end{matrix} \sim \mathcal{N}(0, \sigma \mathbf{I})$$

Leaves 2 **free** components we can sample from.

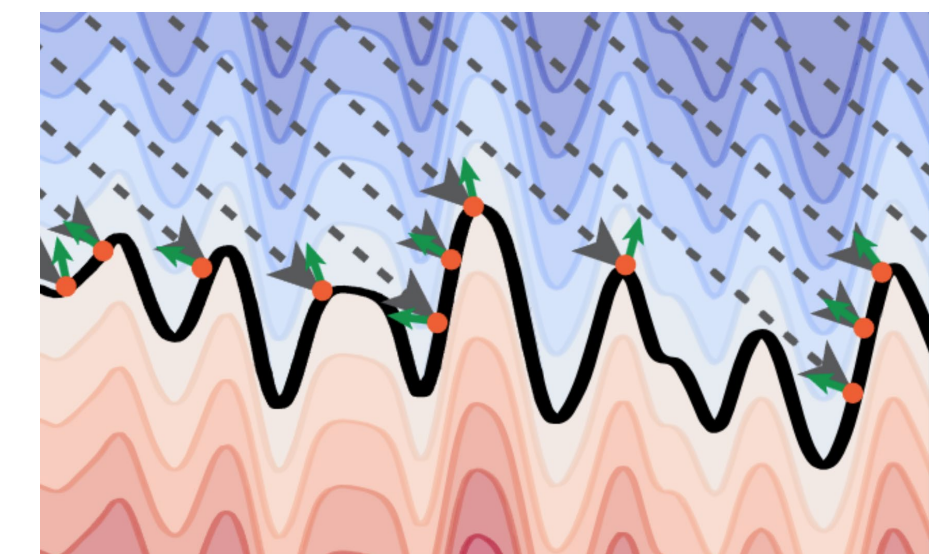
Ensemble-Averaged Light Transport



Conditional Visible
Gradient Distribution
Function (cvGDF)

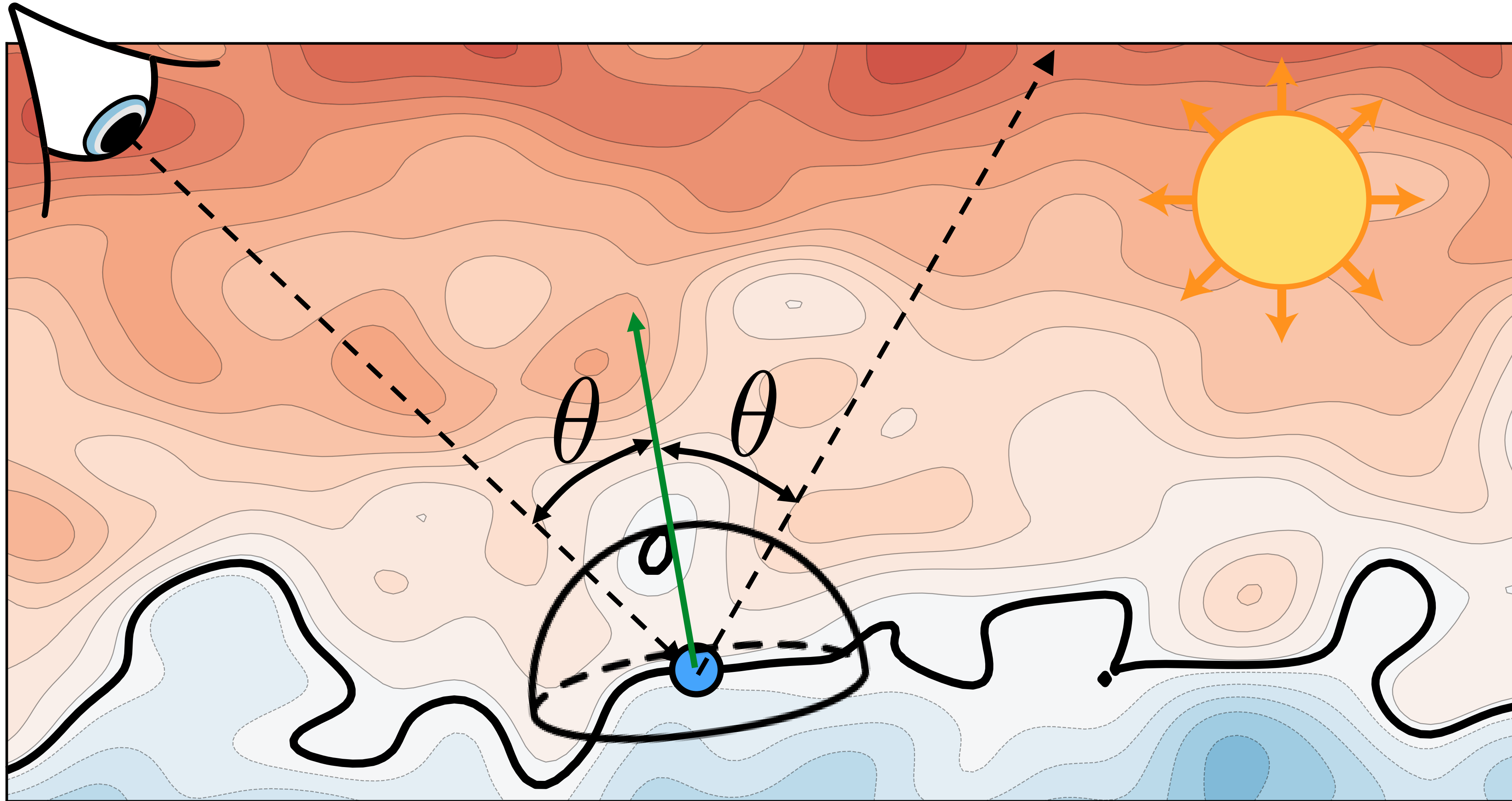


Ray marching filters visible gradients

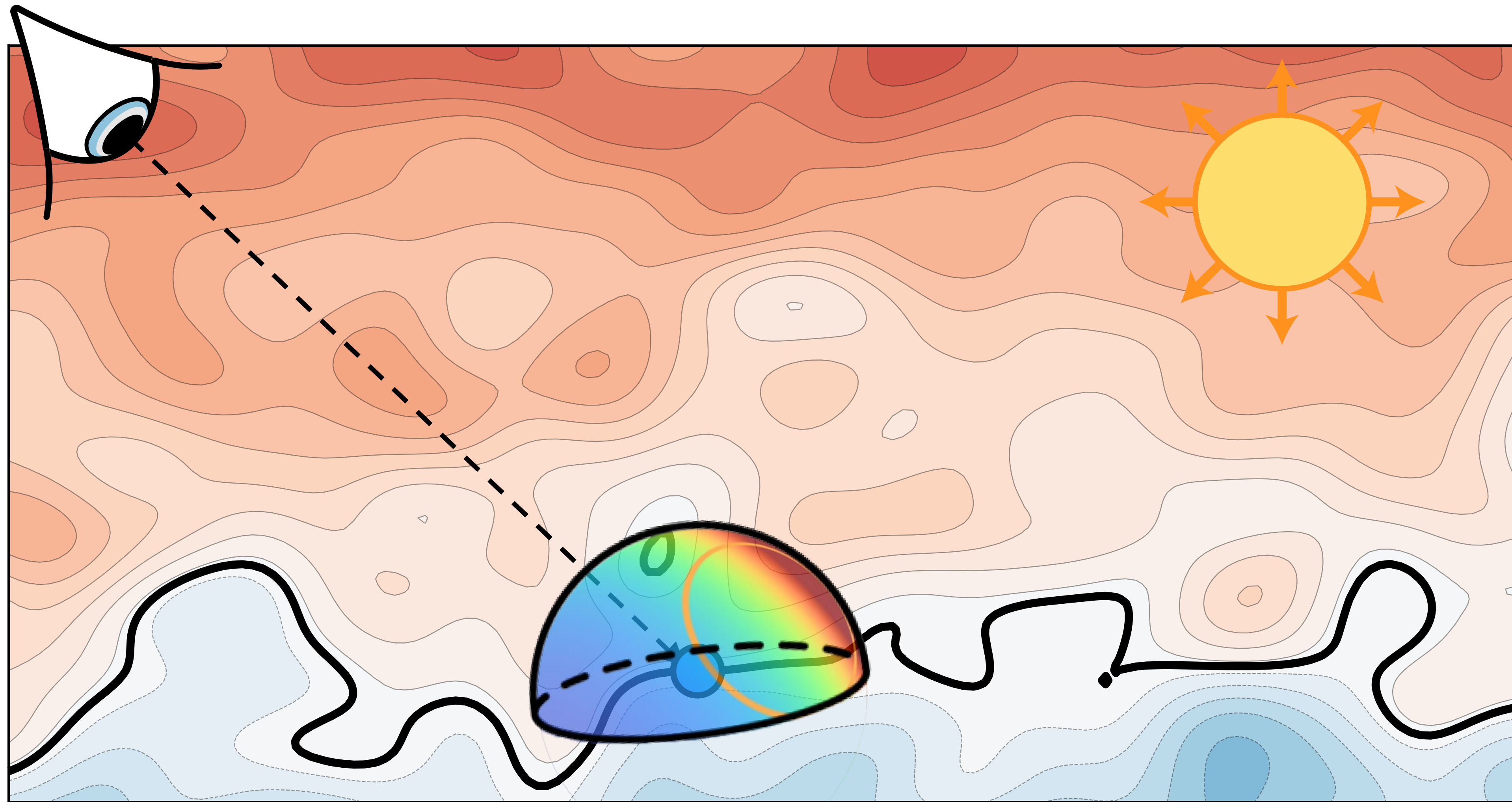


[Seyb et al. 2024]

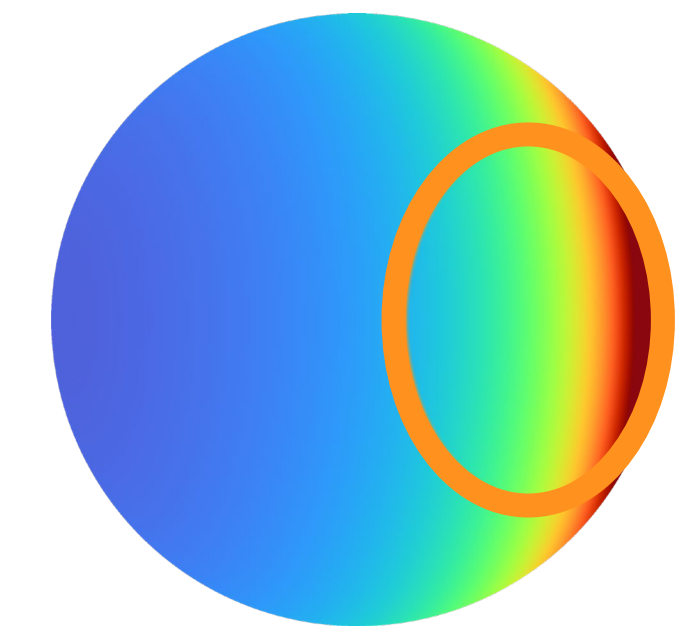
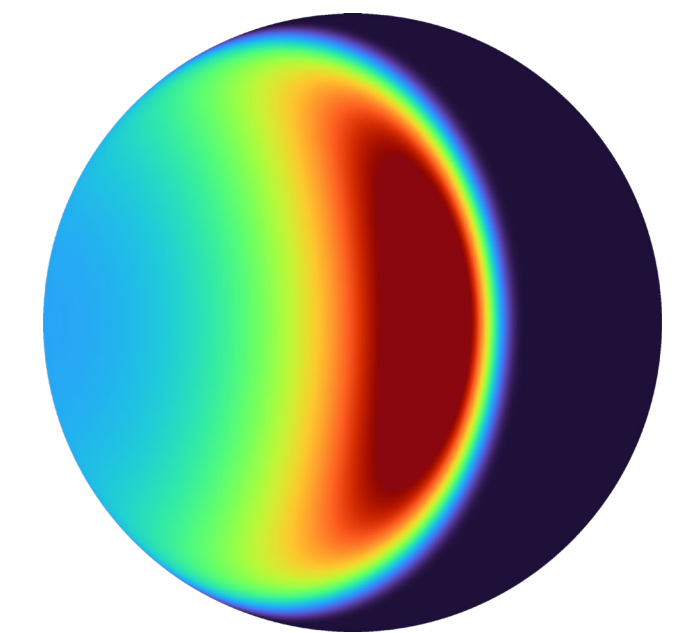
Ensemble-Averaged Light Transport



Ensemble-Averaged Light Transport

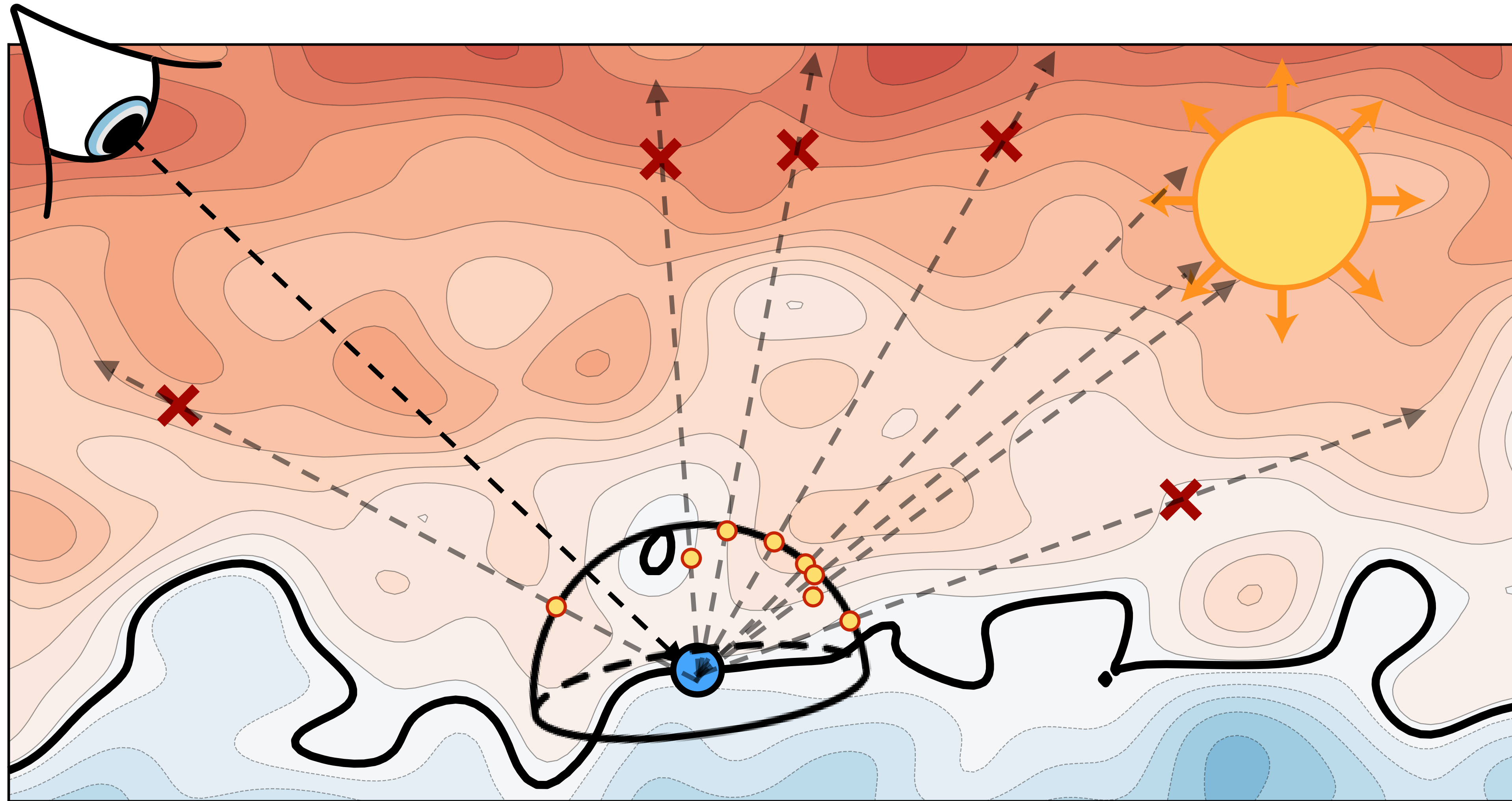


Conditional Visible
Gradient Distribution
Function (cvGDF)

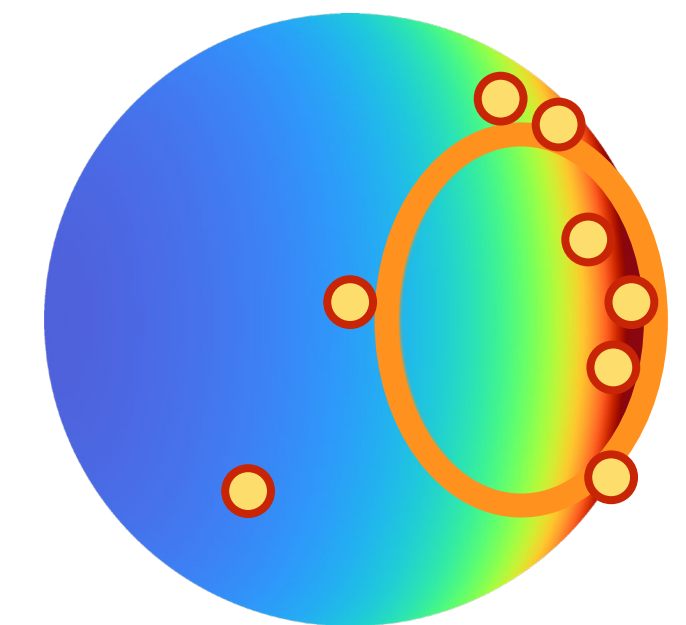
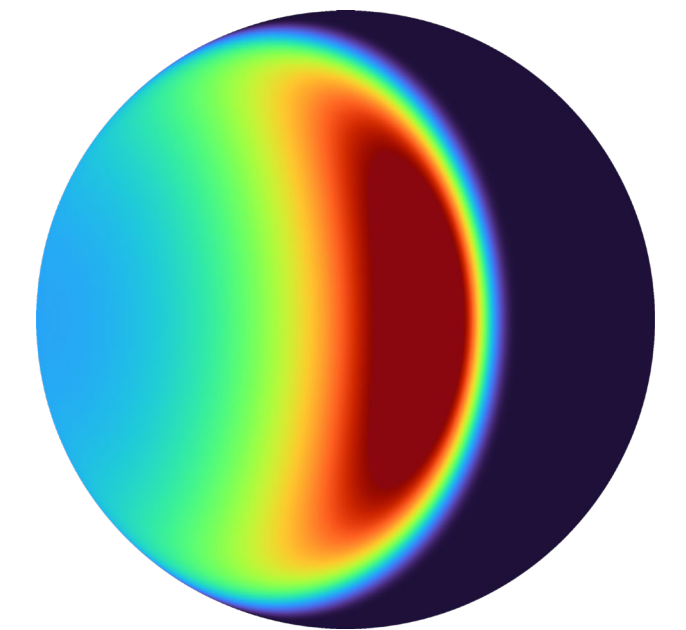


*Conditional Scattering
Distribution Function*

Ensemble-Averaged Light Transport – Unidirectional Sampling



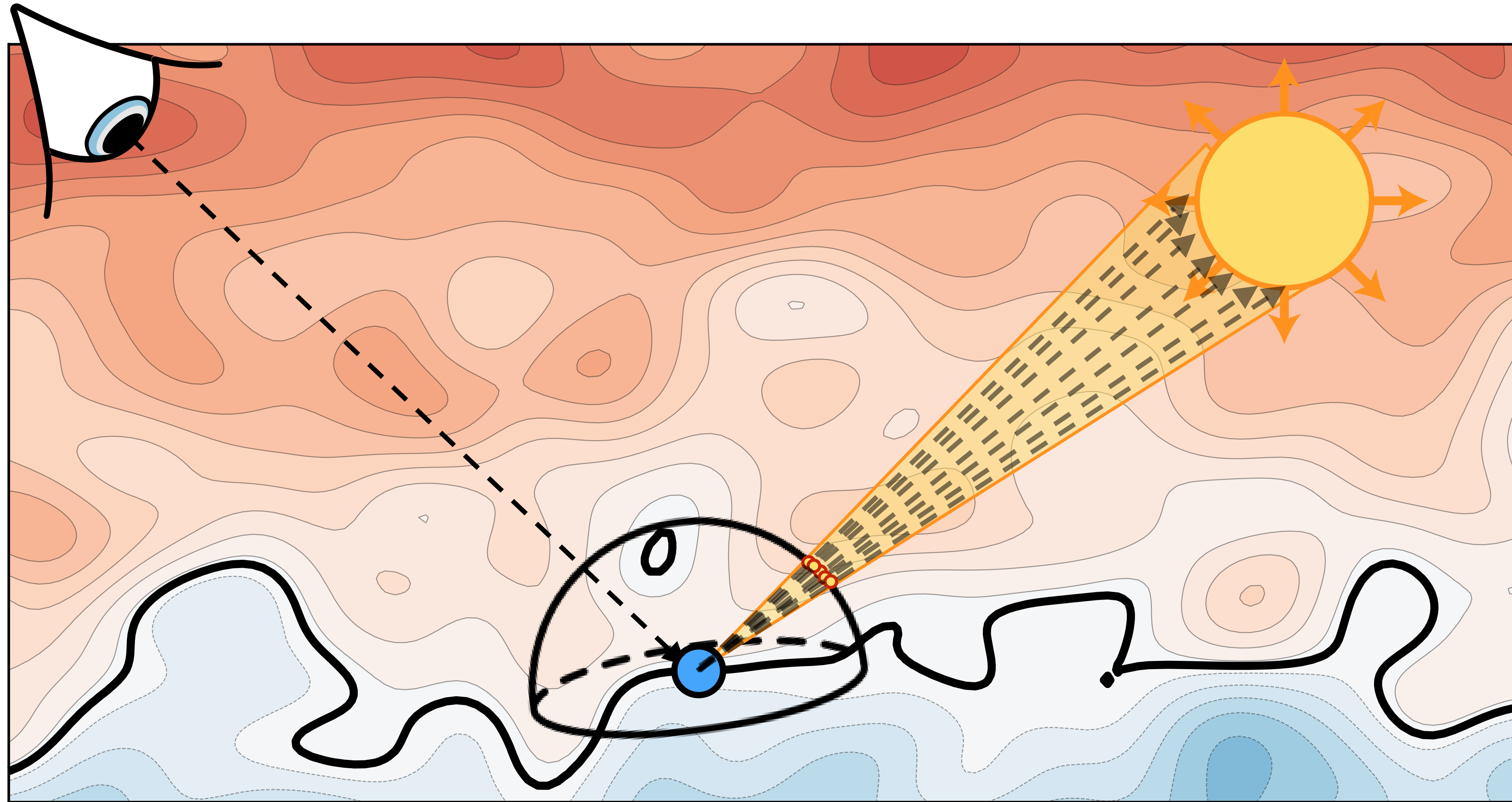
Conditional Visible
Gradient Distribution
Function (cvGDF)



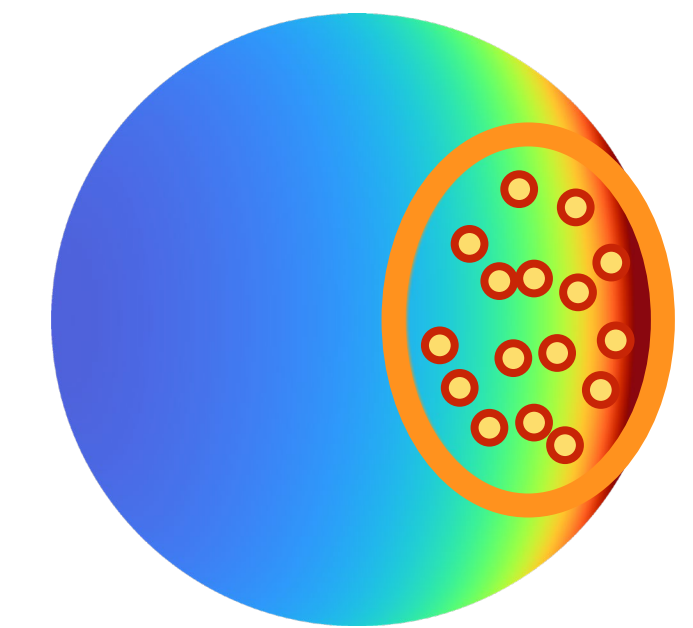
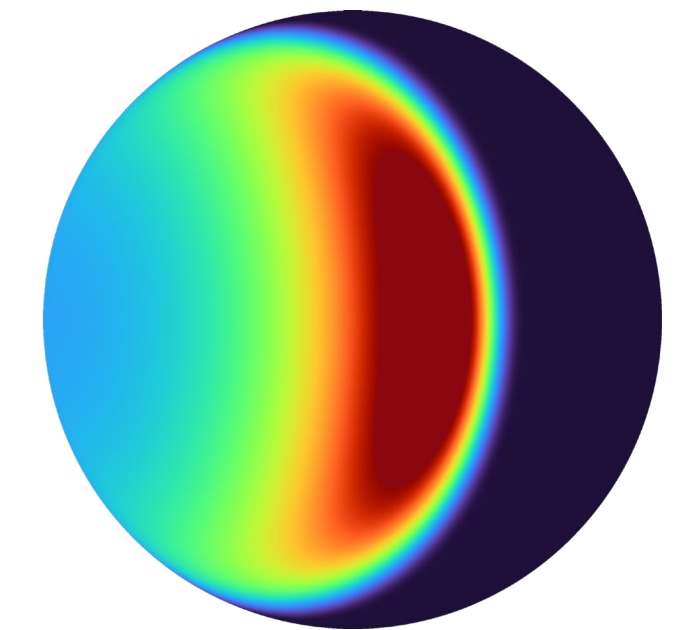
*Conditional Scattering
Distribution Function*

Ensemble-Averaged Light Transport – Light Sampling

(next event estimation)

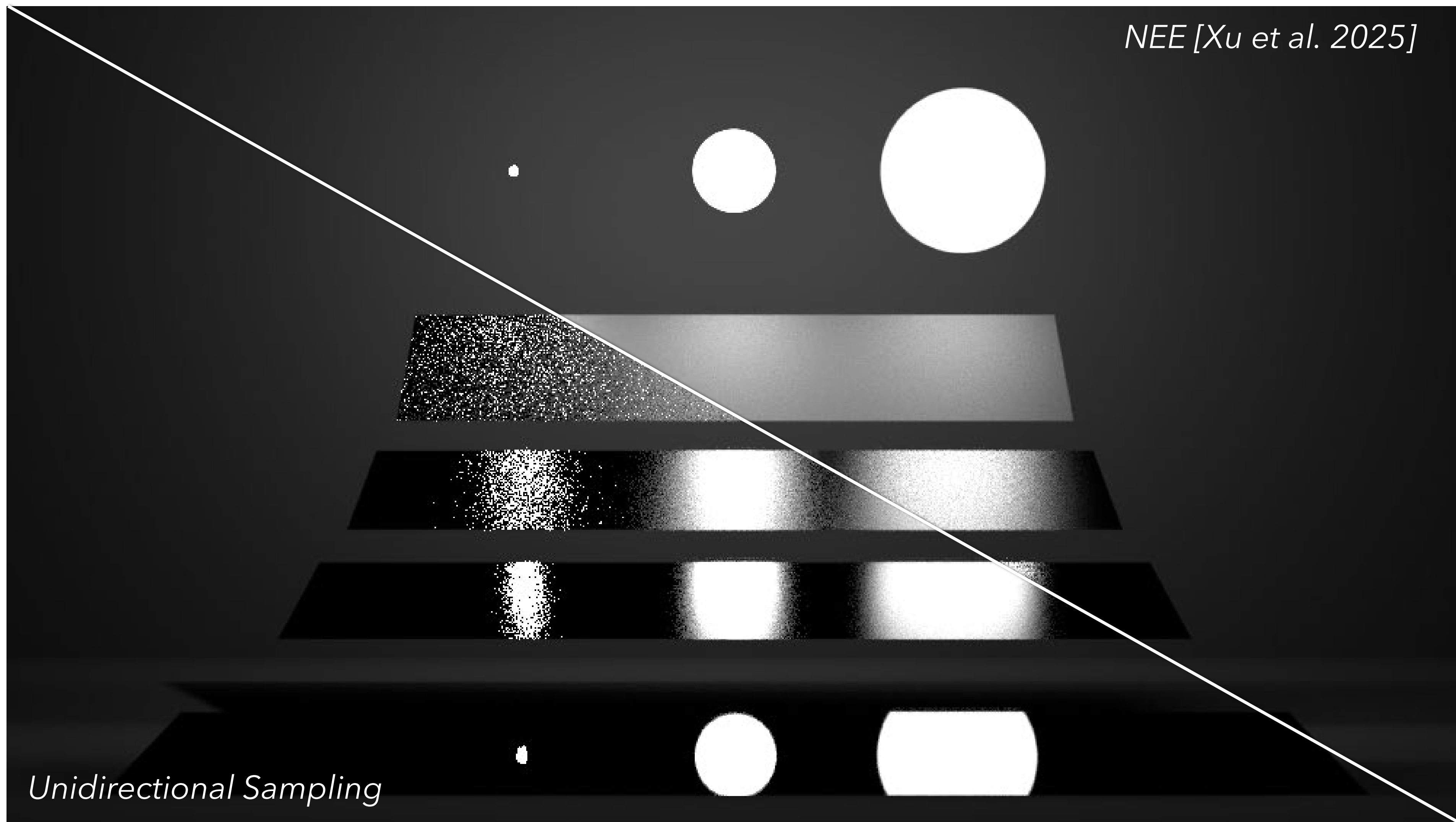


Conditional Visible
Gradient Distribution
Function (cvGDF)



*Conditional Scattering
Distribution Function*

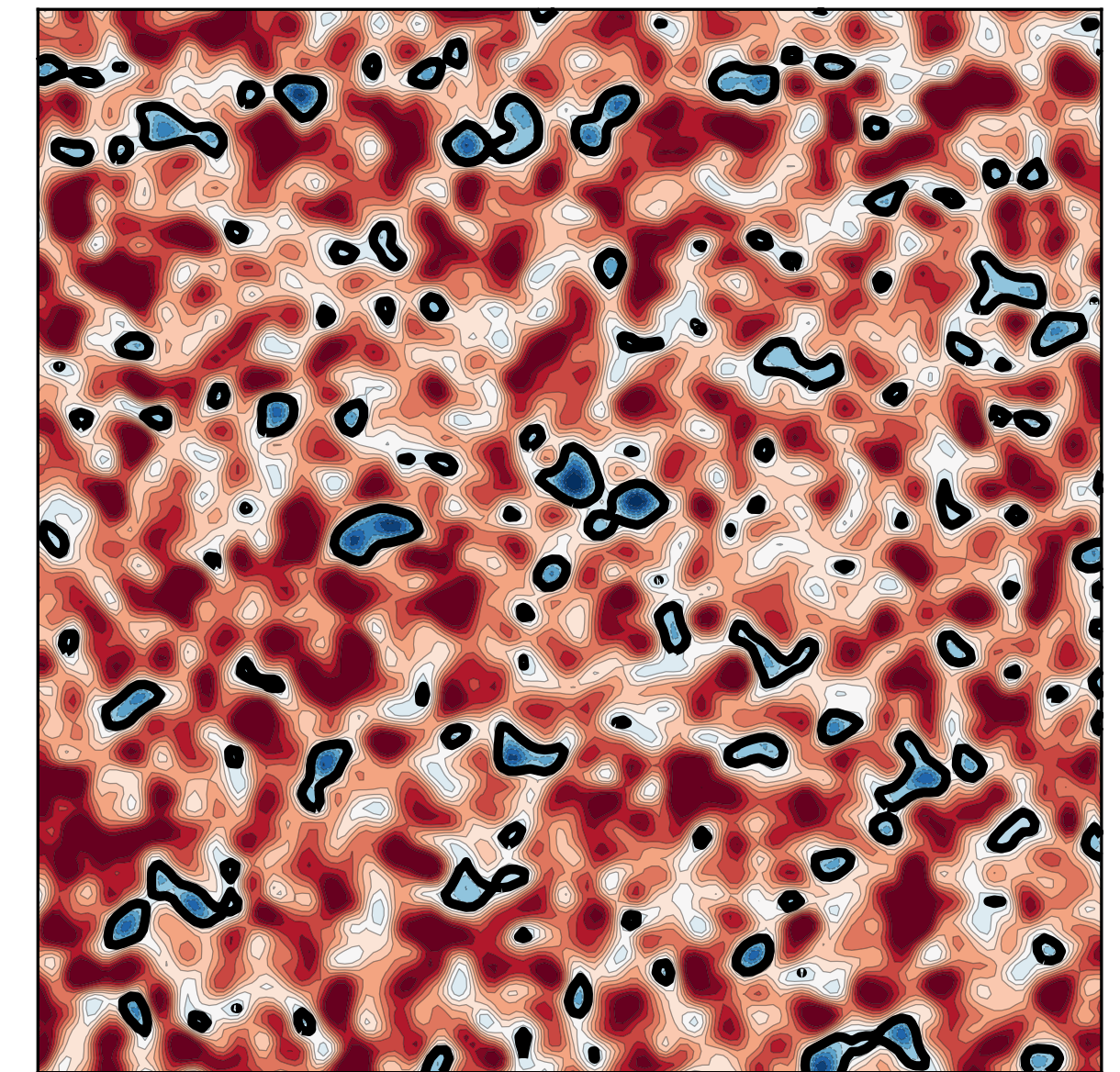
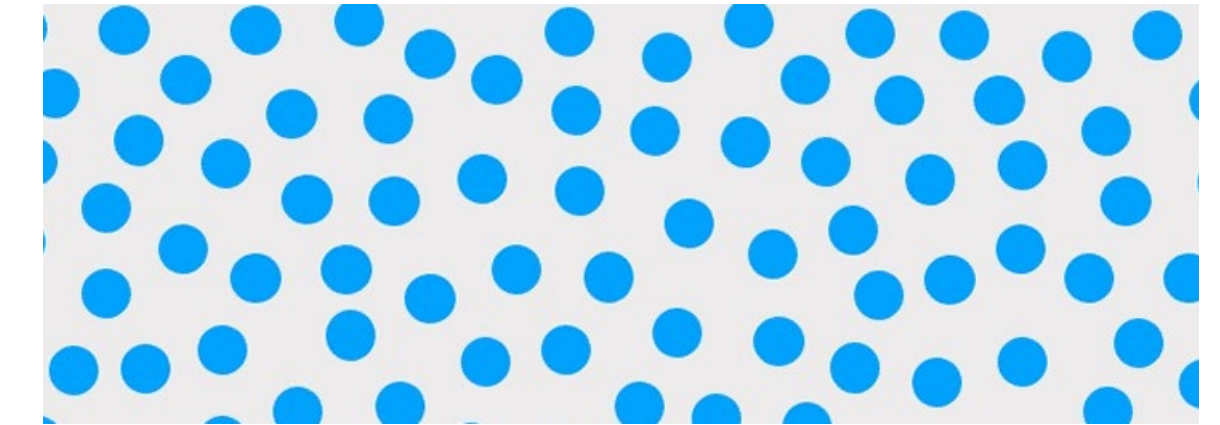
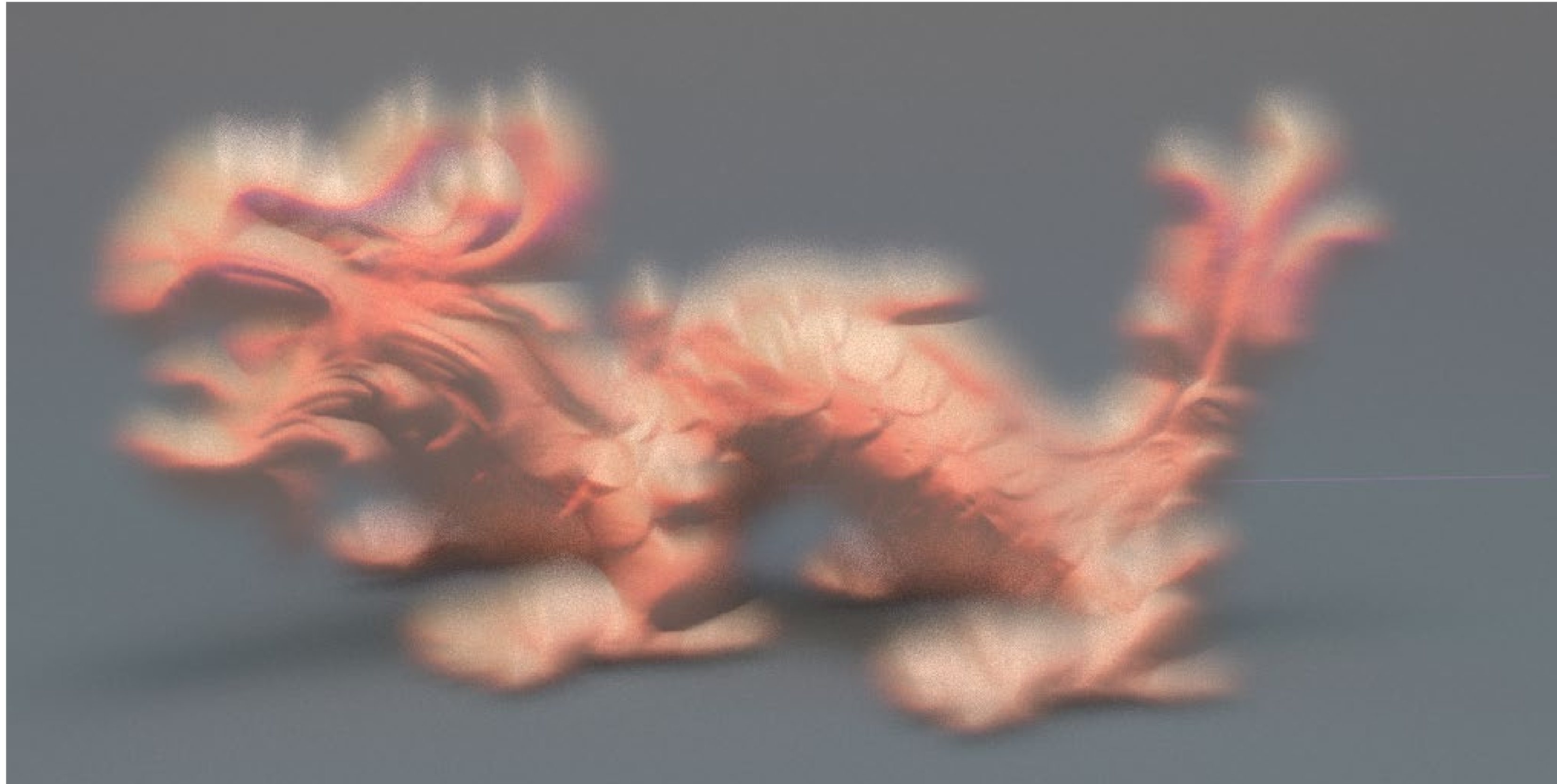
UNI vs NEE





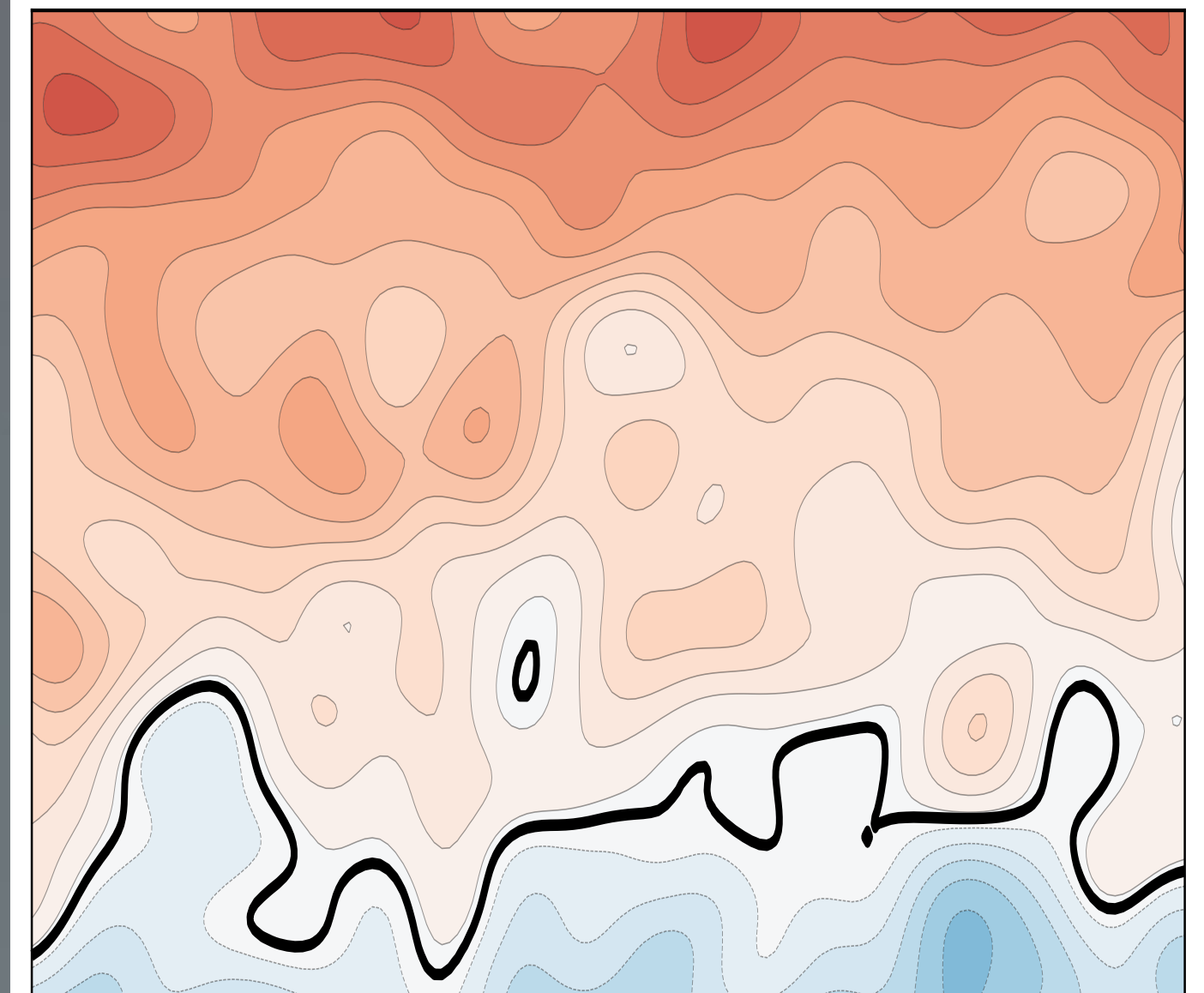
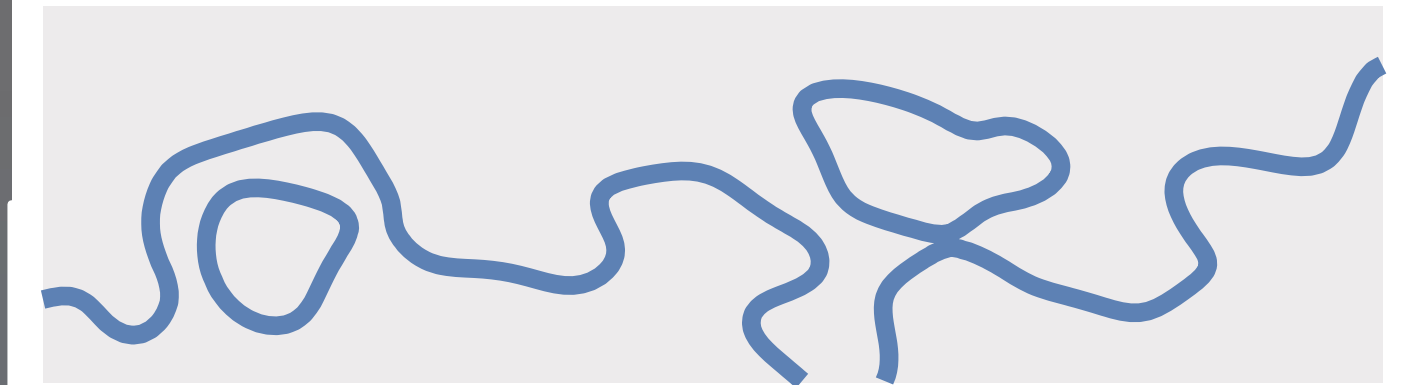
Problem Analysis & Motivation

Problem Statement



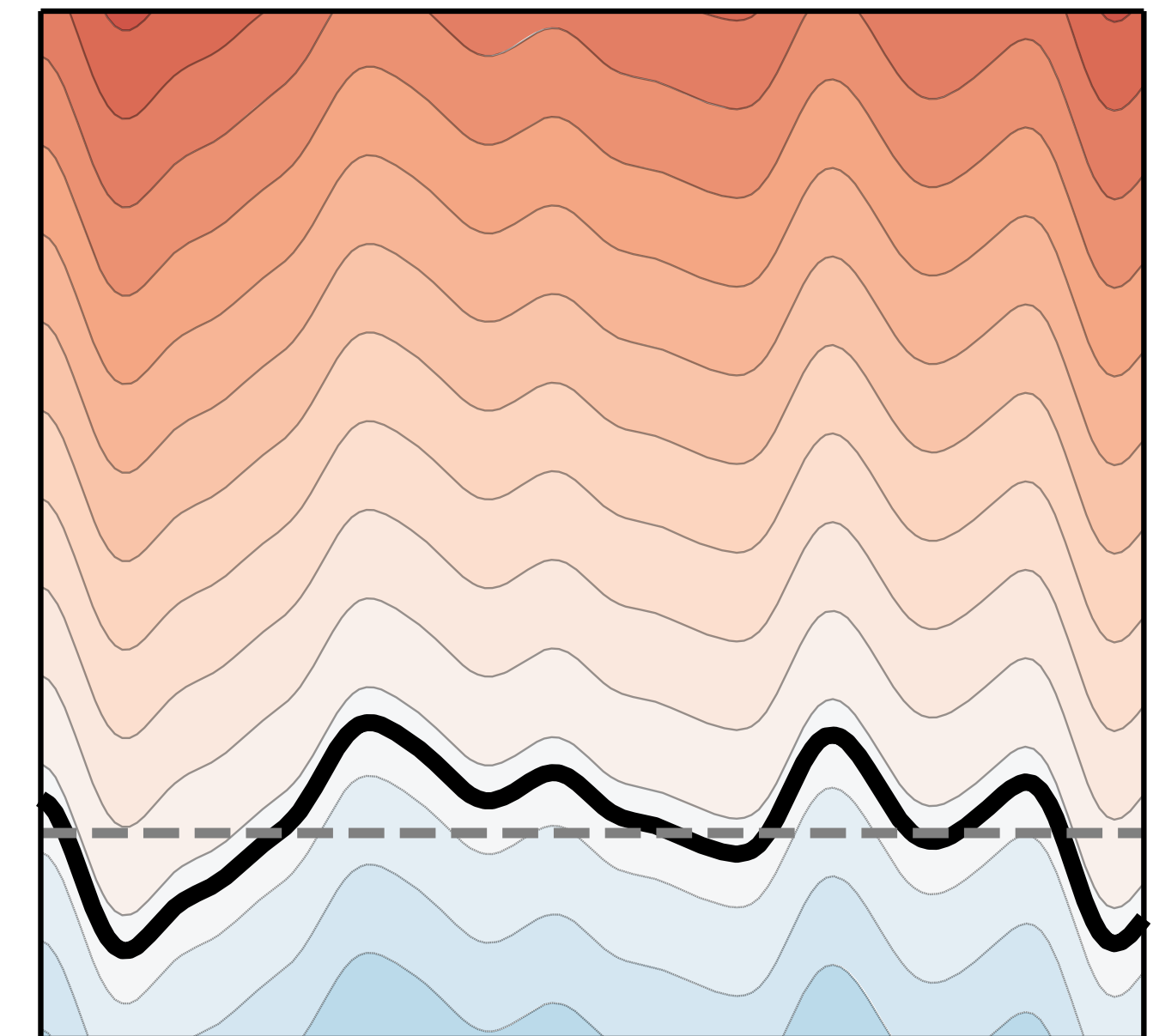
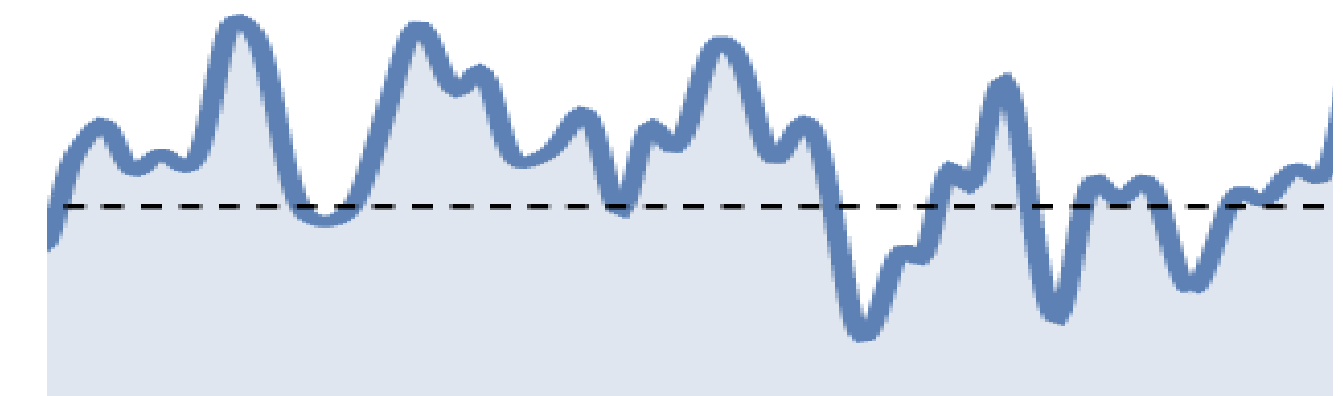
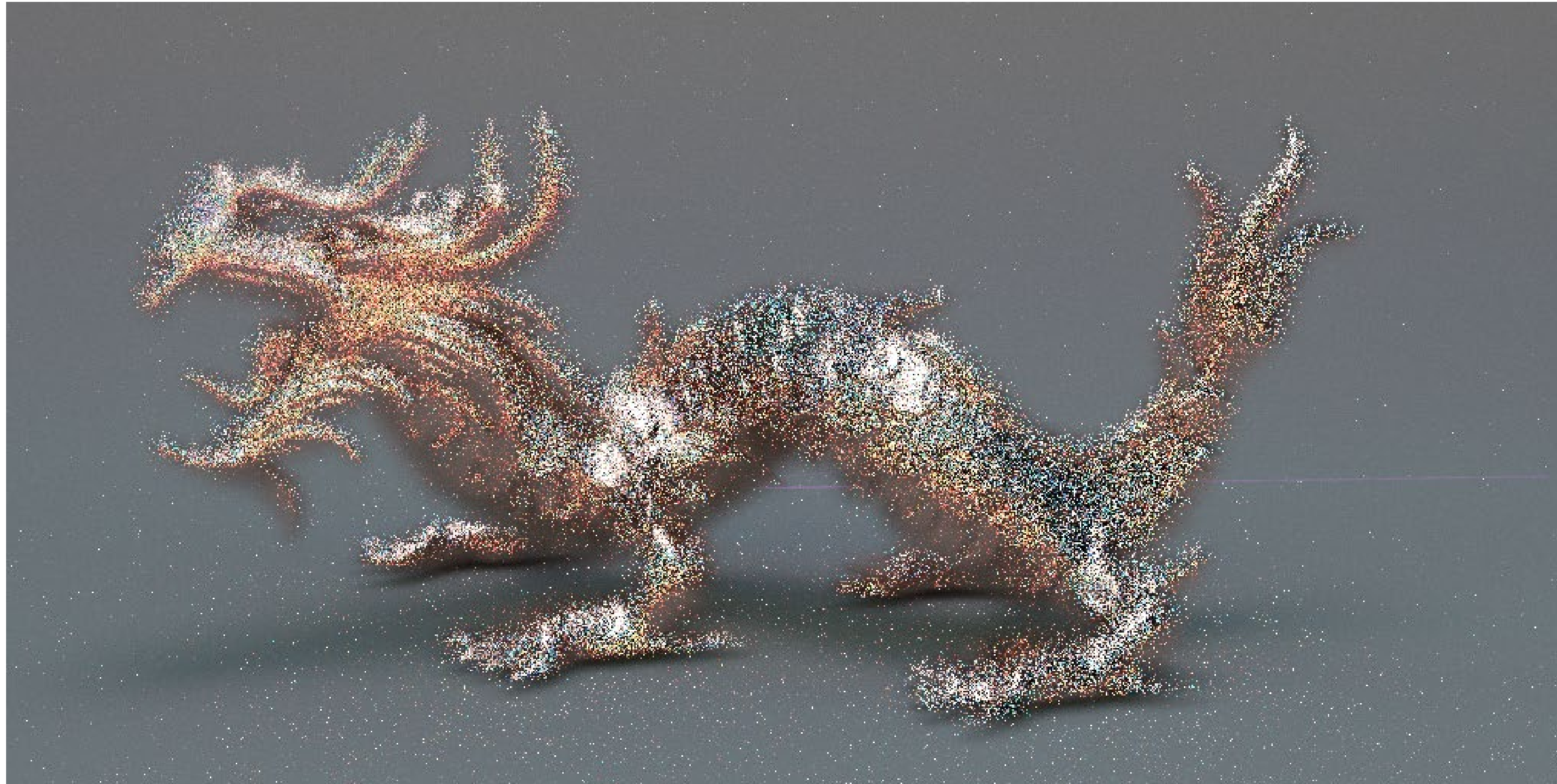
Volume-like

Problem Statement



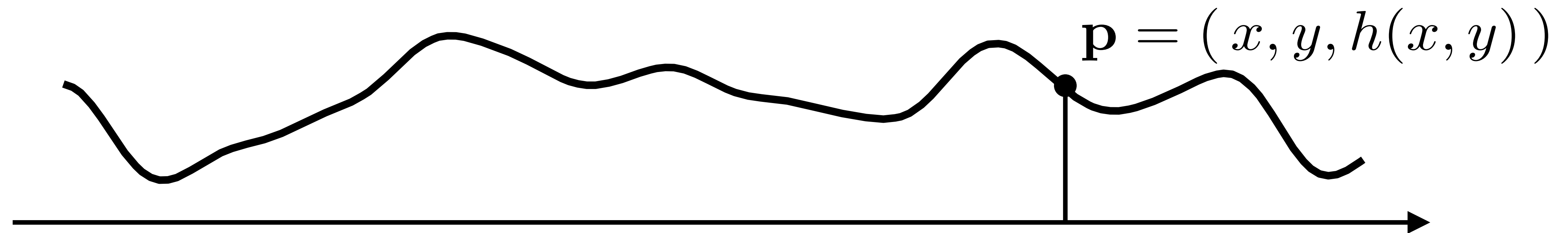
In-between

Problem Statement

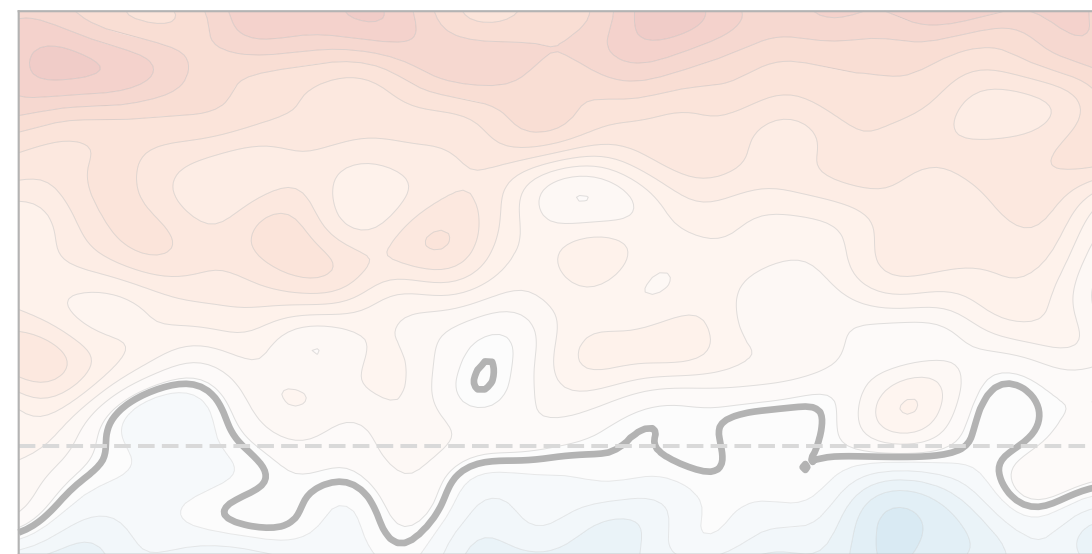


Surface-like

Heightfields



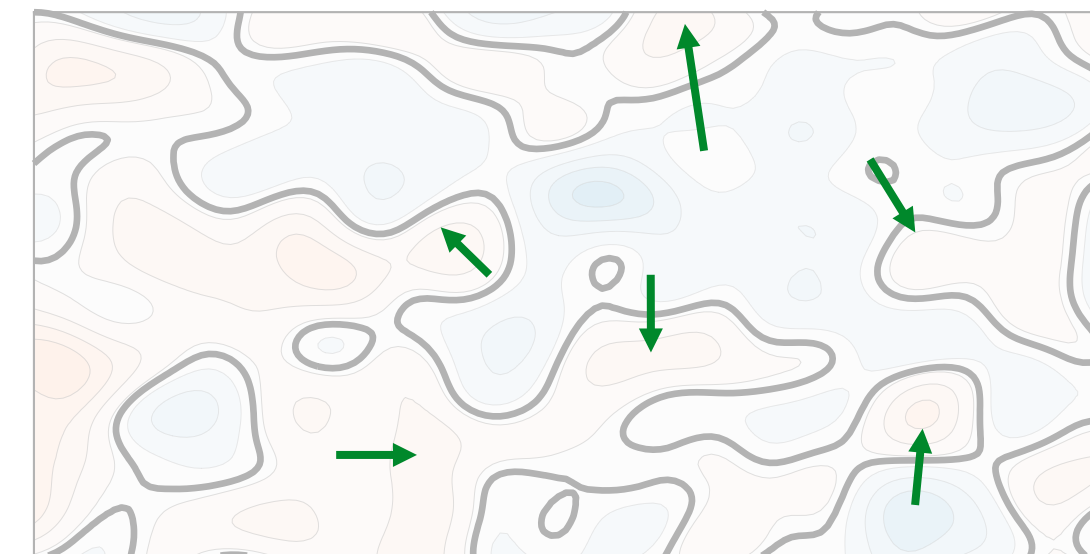
Gradient distribution of Heightfield GPISeS



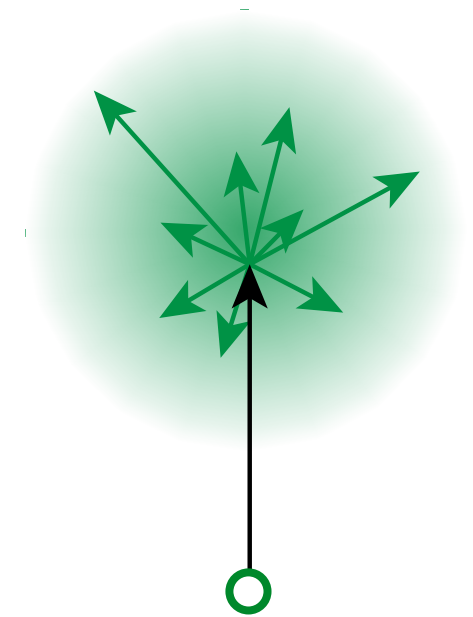
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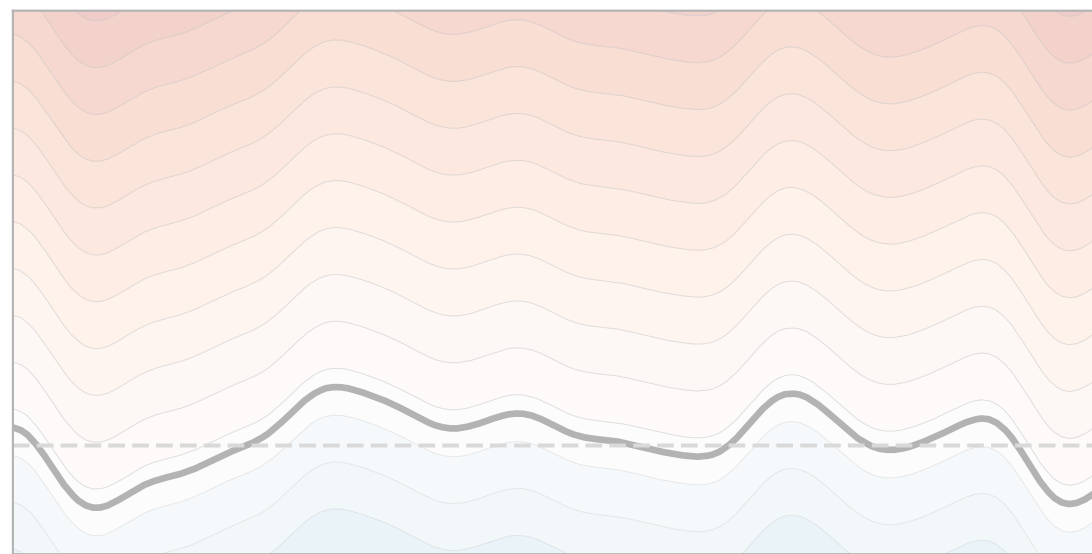
+



*Gradient can point
-up/down
-left/right*



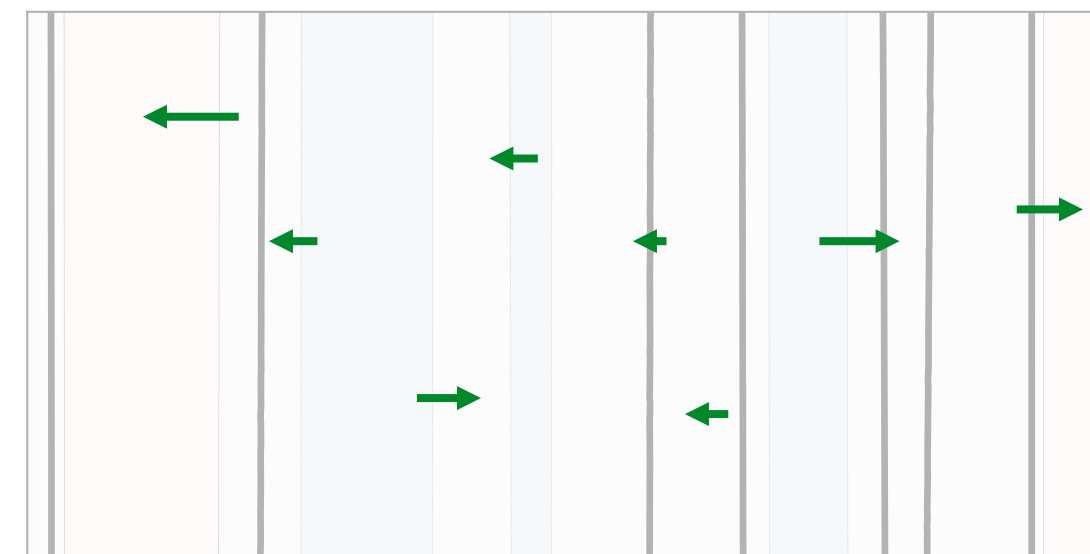
Lose 1 DoF in gradient when stretching the noise



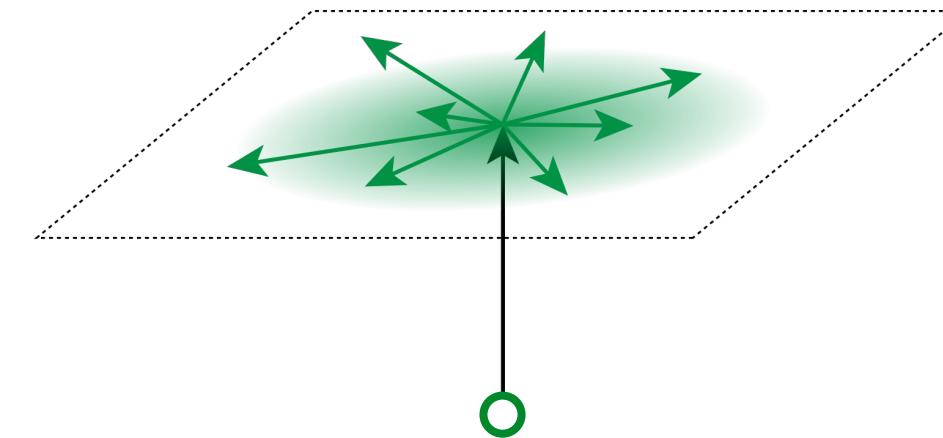
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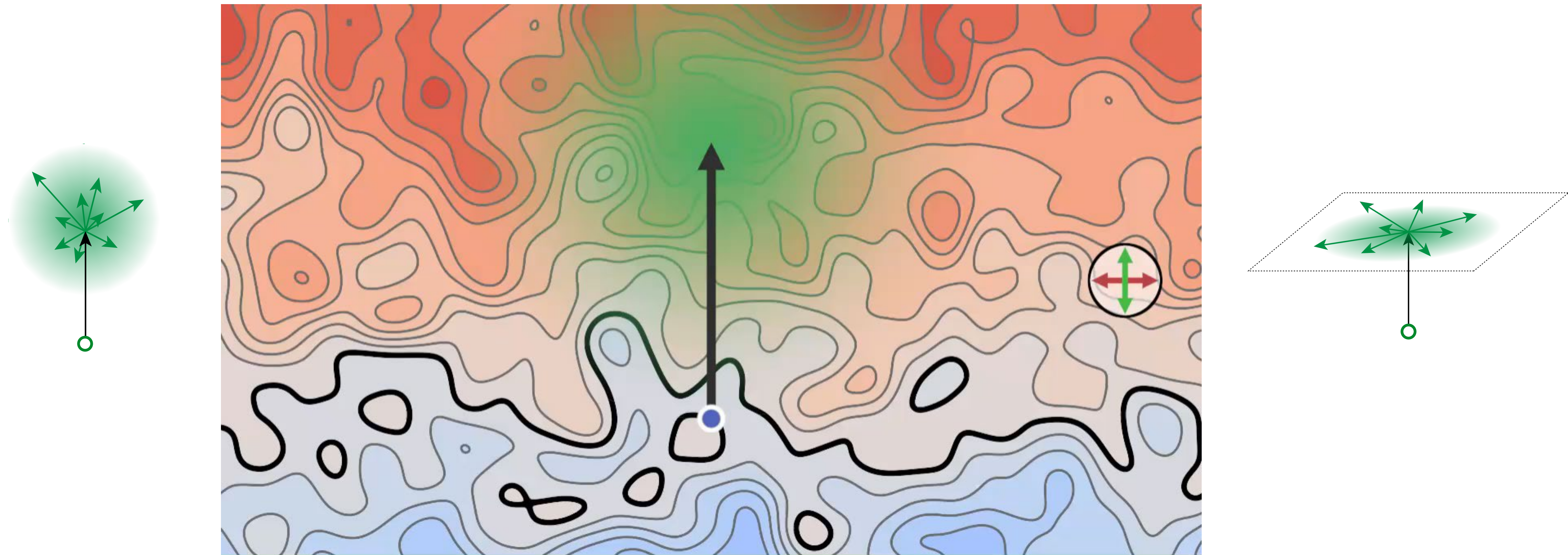
+



*Gradient can only point
-left/right*

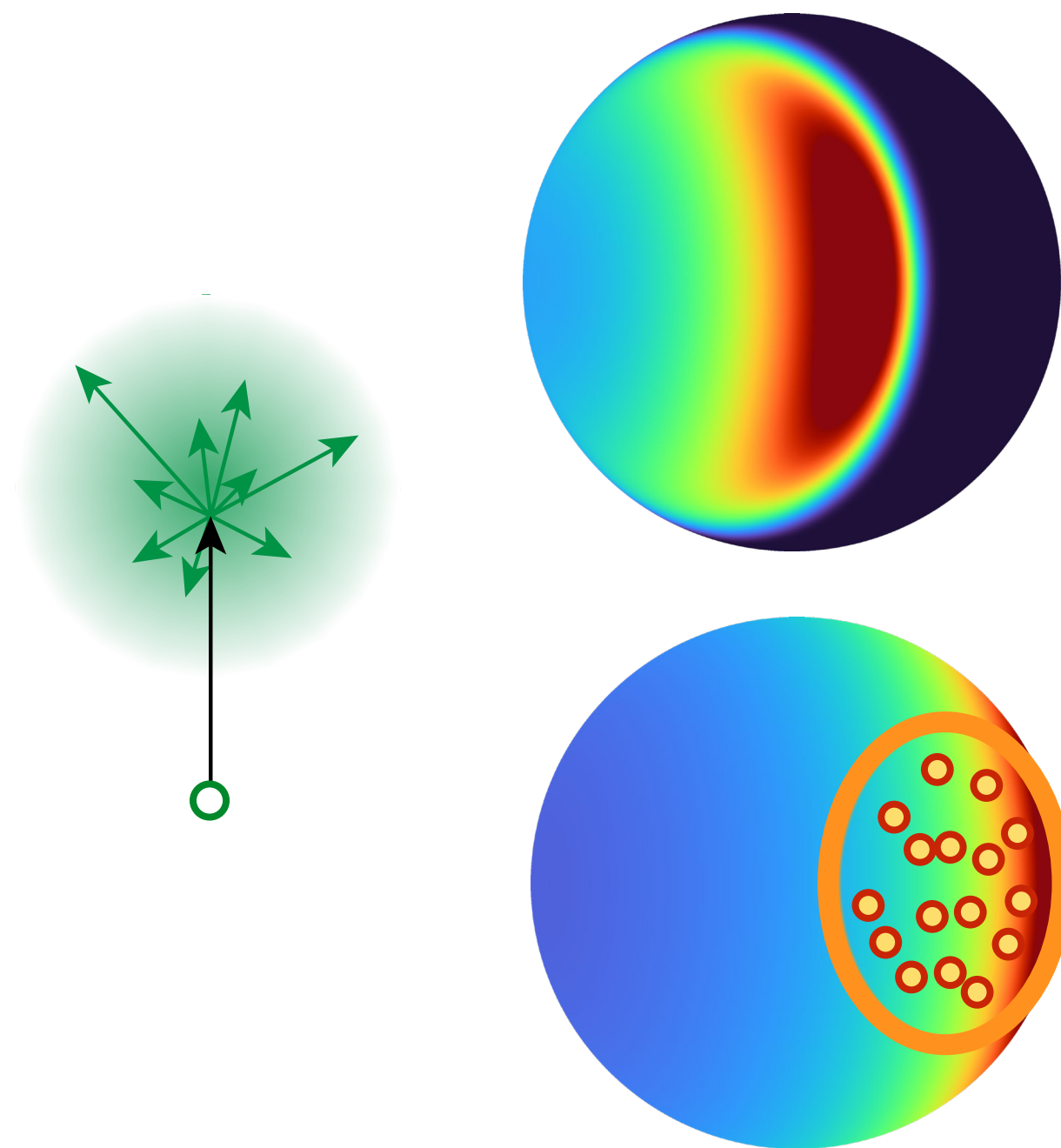


Gradient distribution of Heightfield GPISes

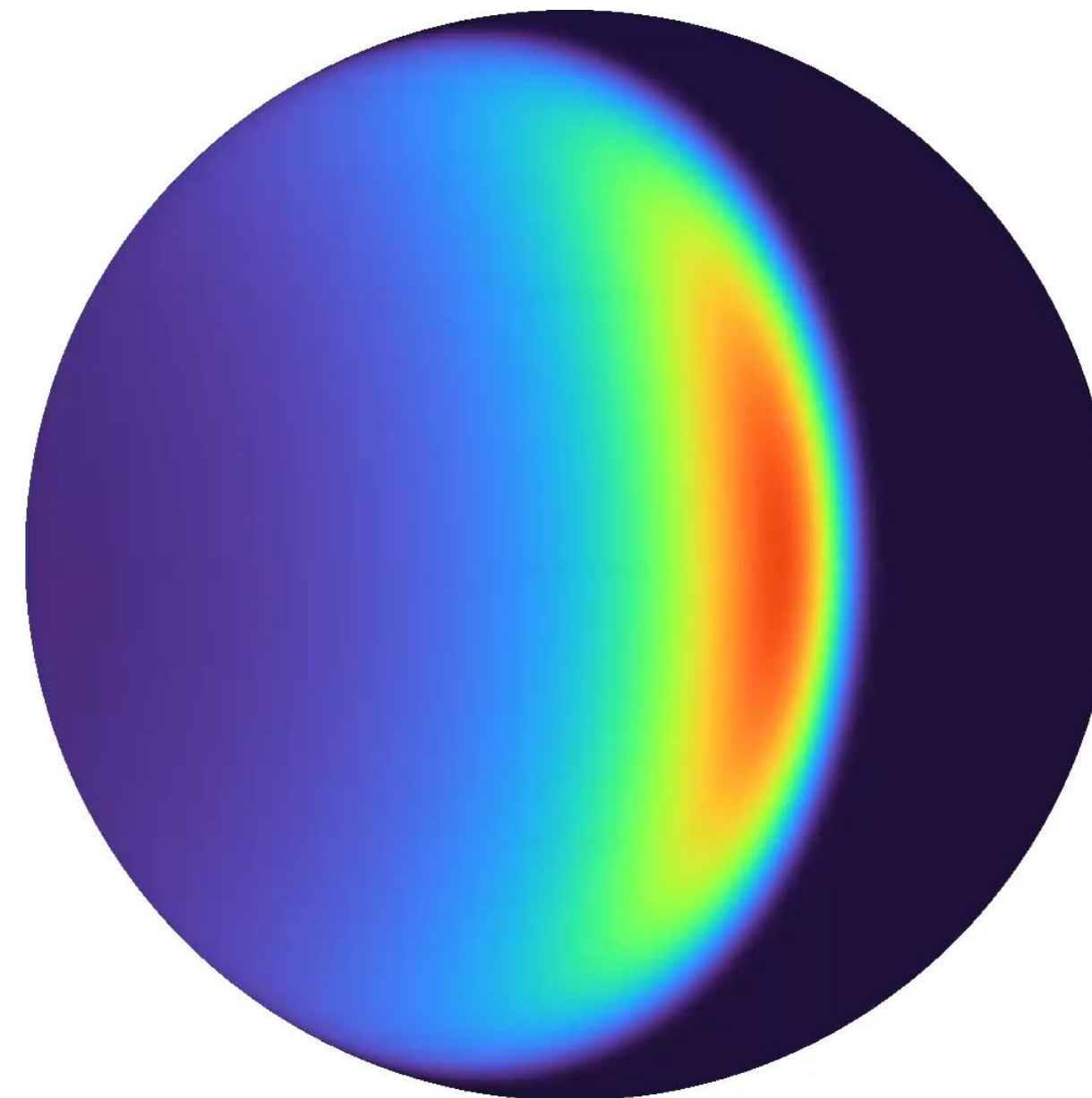


NEE for heightfield GPIS

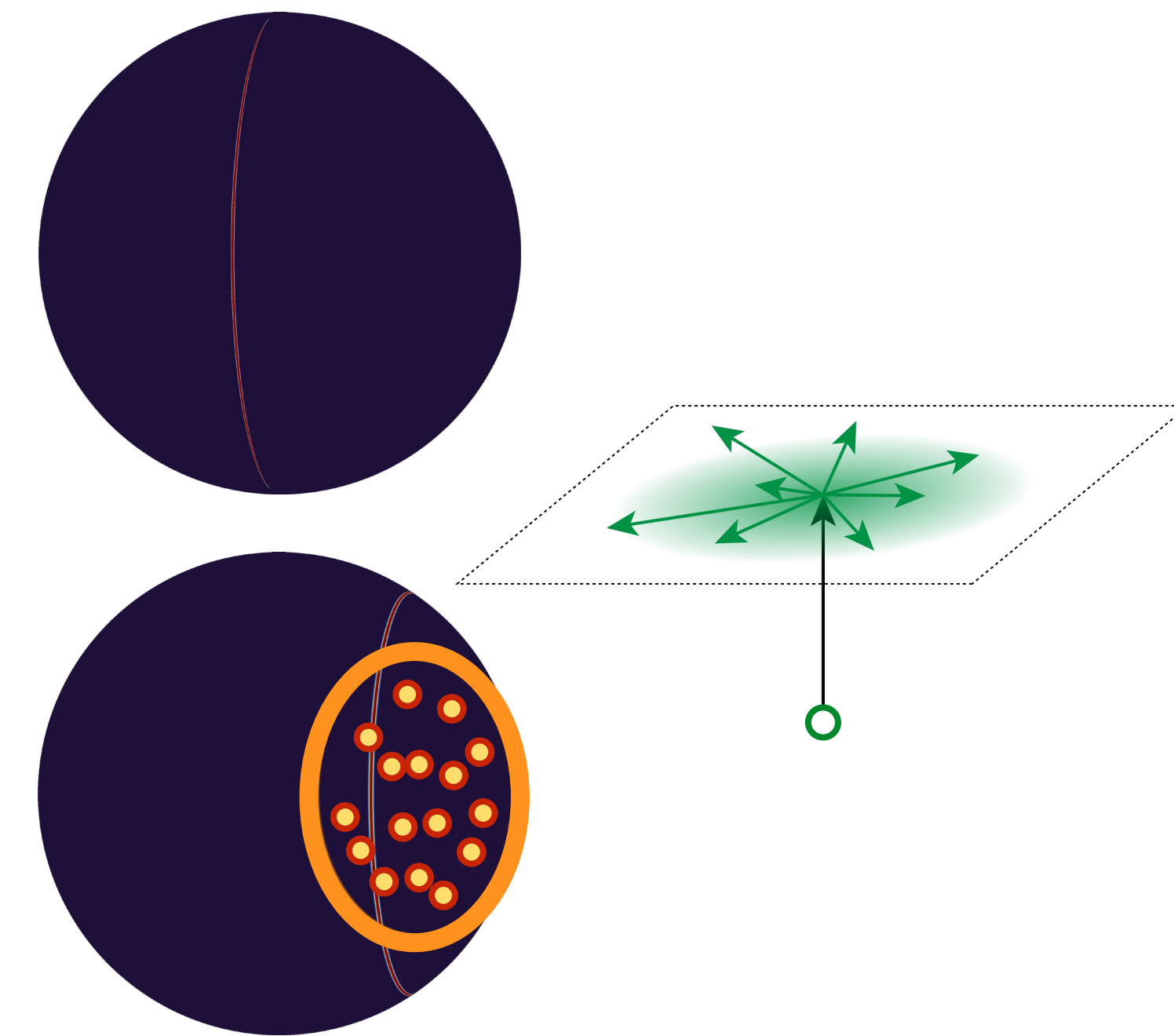
Conditional Visible
Gradient Distribution
Function (cvGDF)



*Conditional Scattering
Distribution Function*

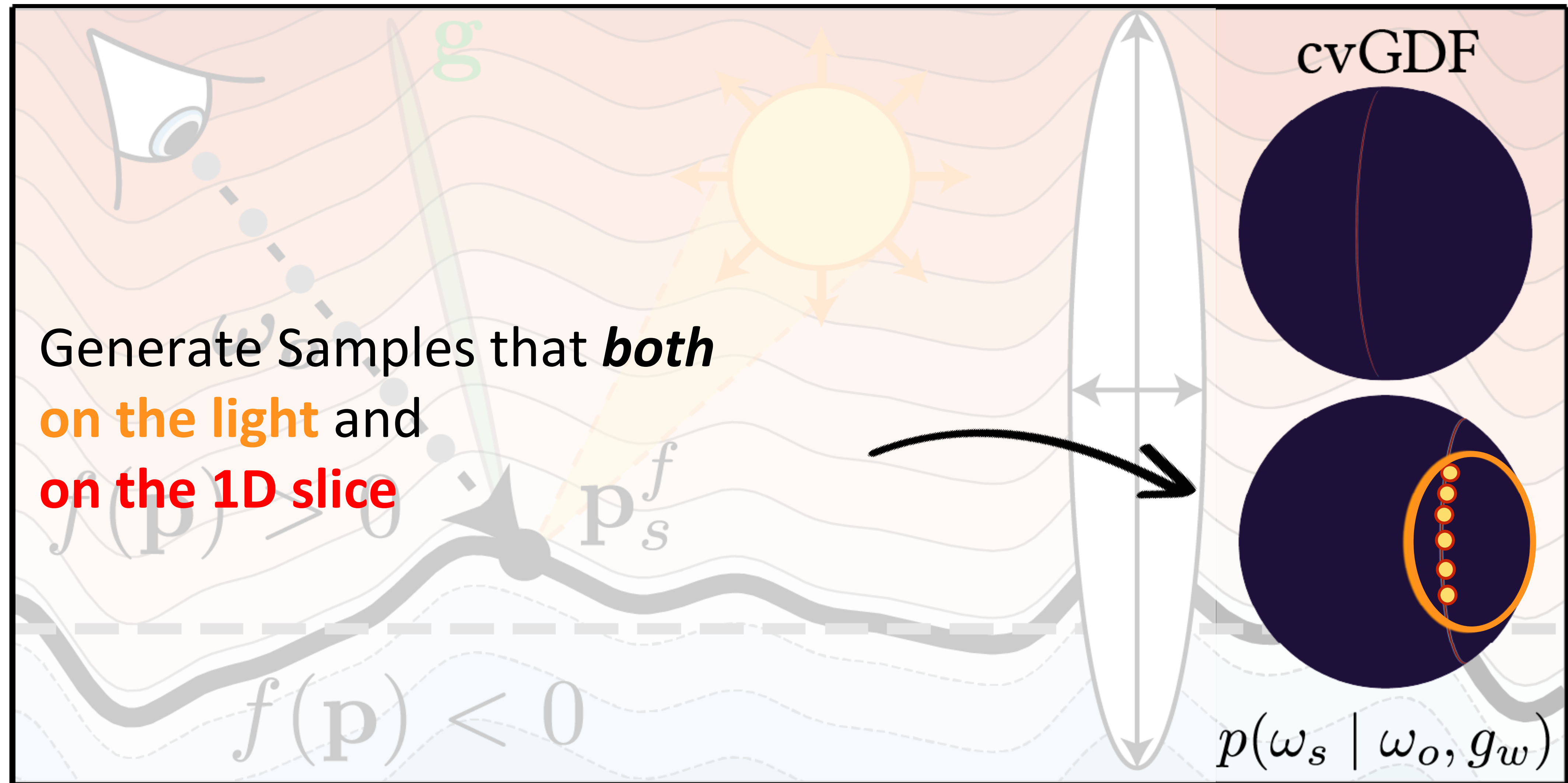


Conditional Visible
Gradient Distribution
Function (cvGDF)



*Conditional Scattering
Distribution Function*

Motivation

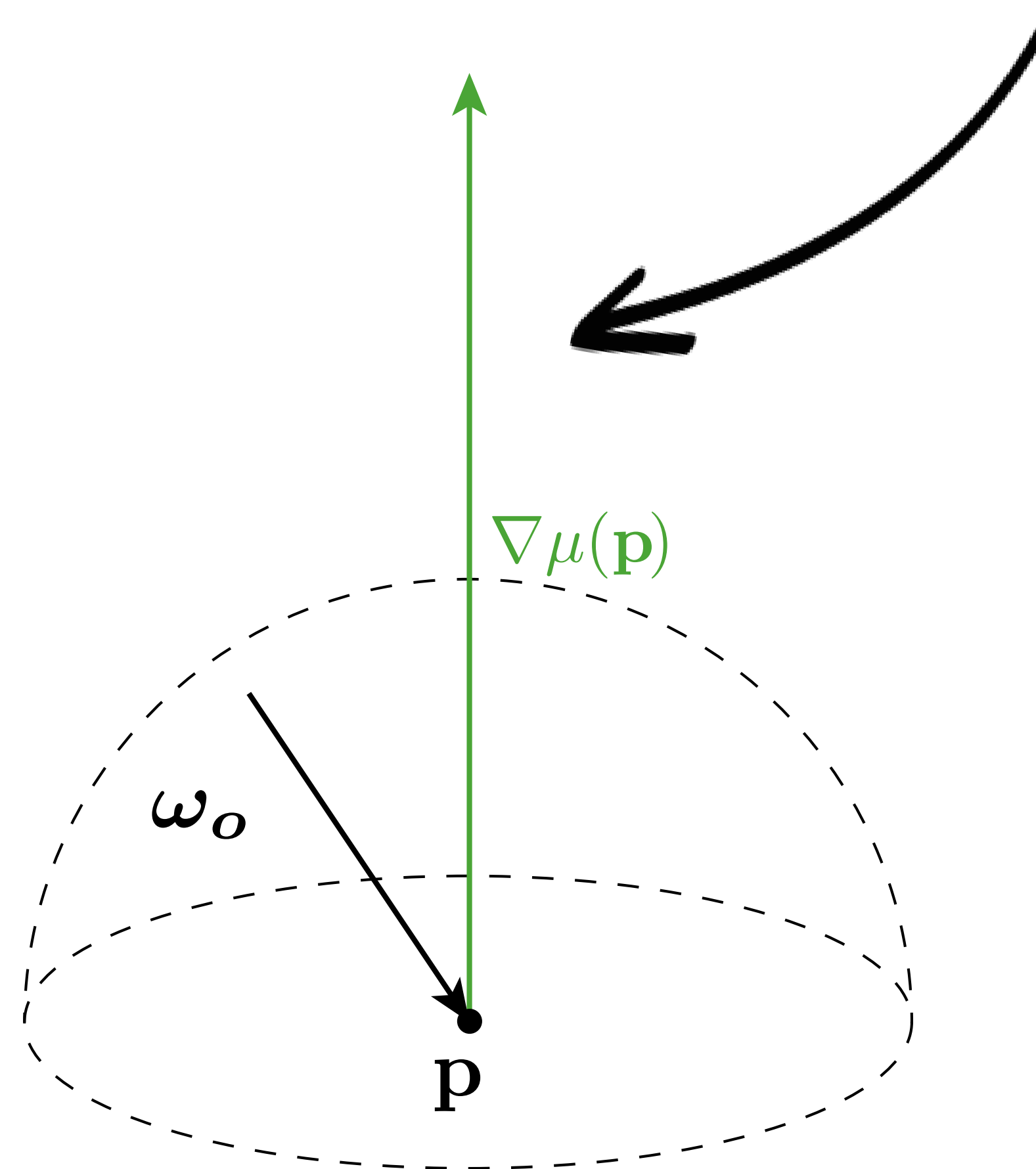




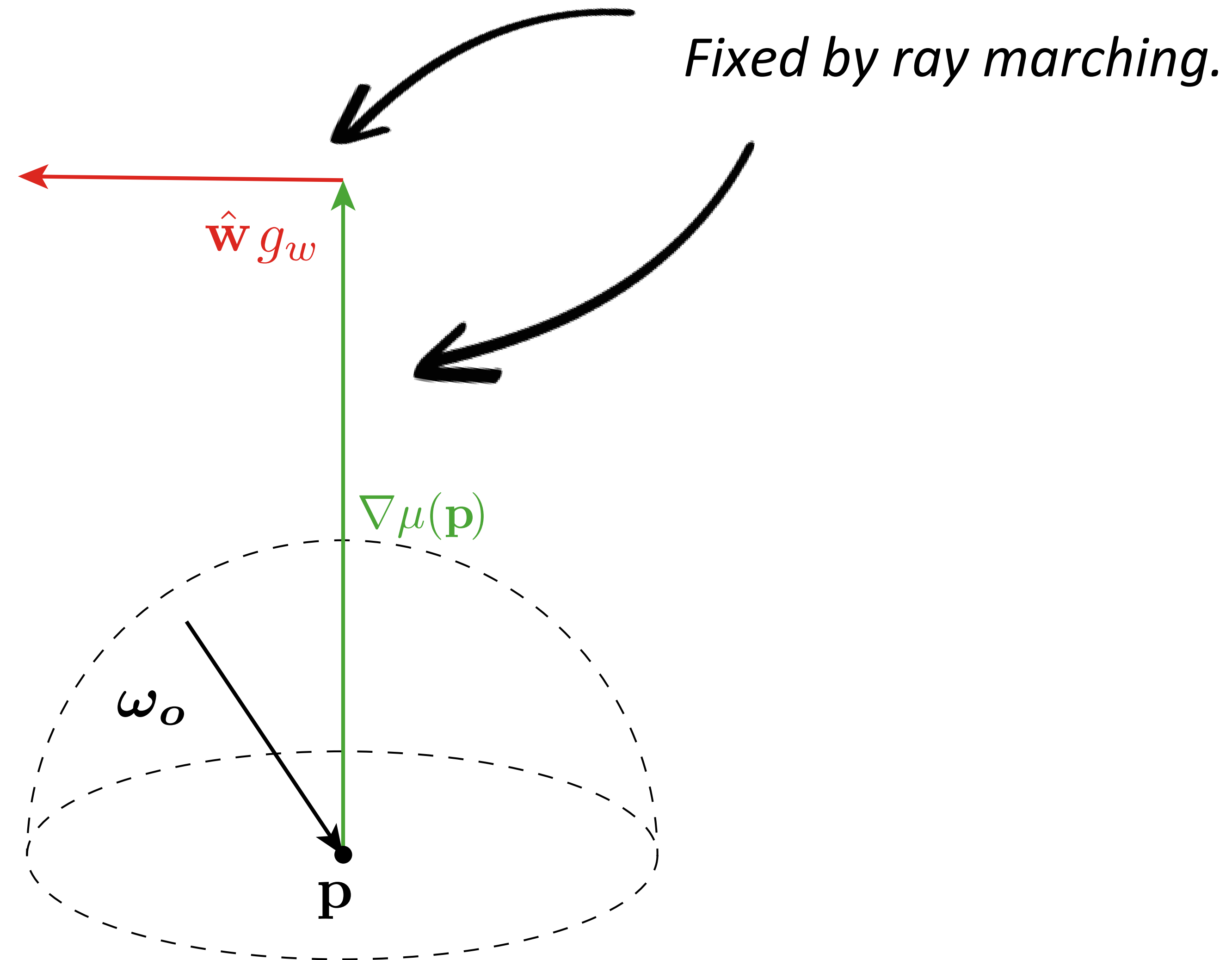
Our Method

Normal Circle

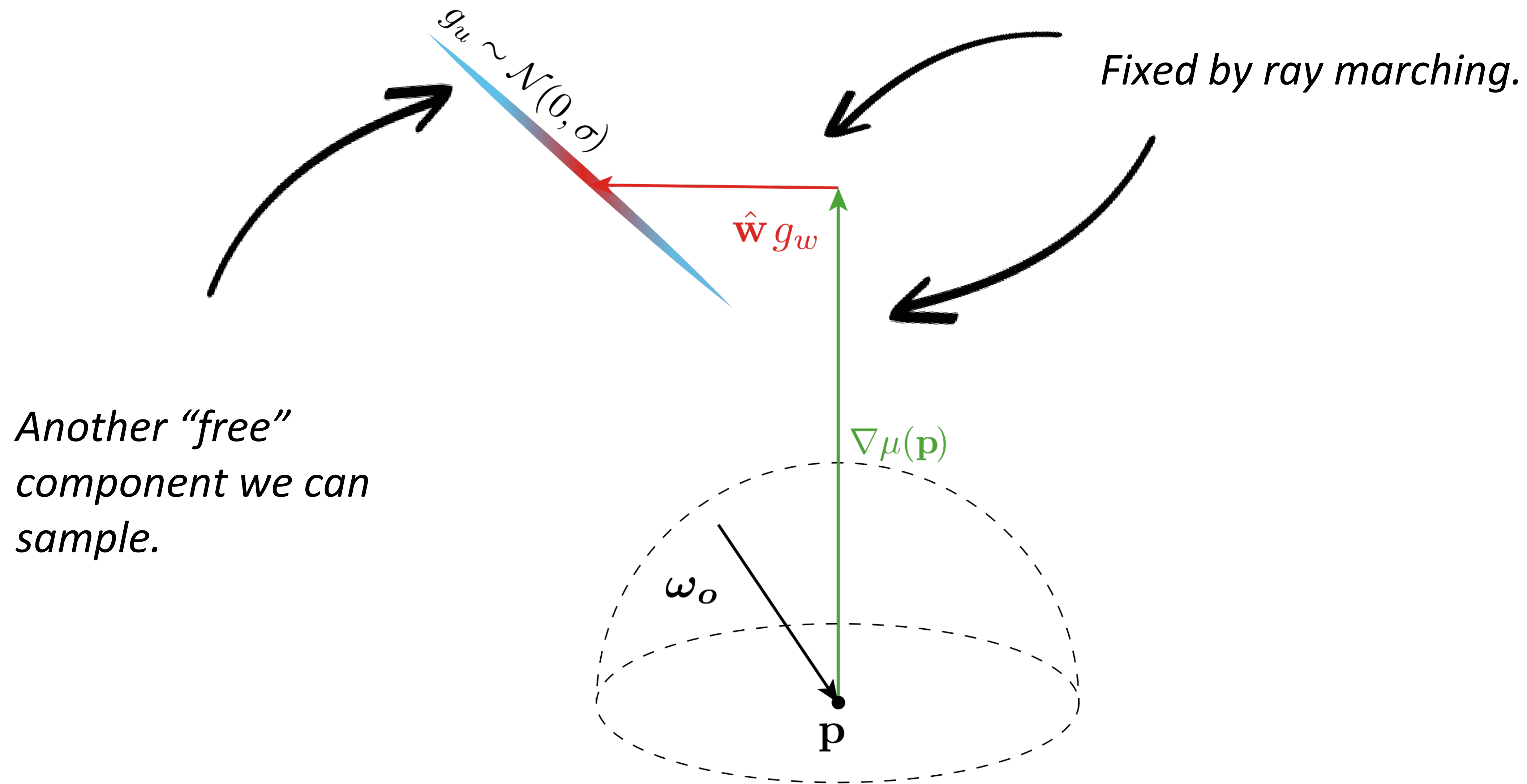
Fixed by ray marching.



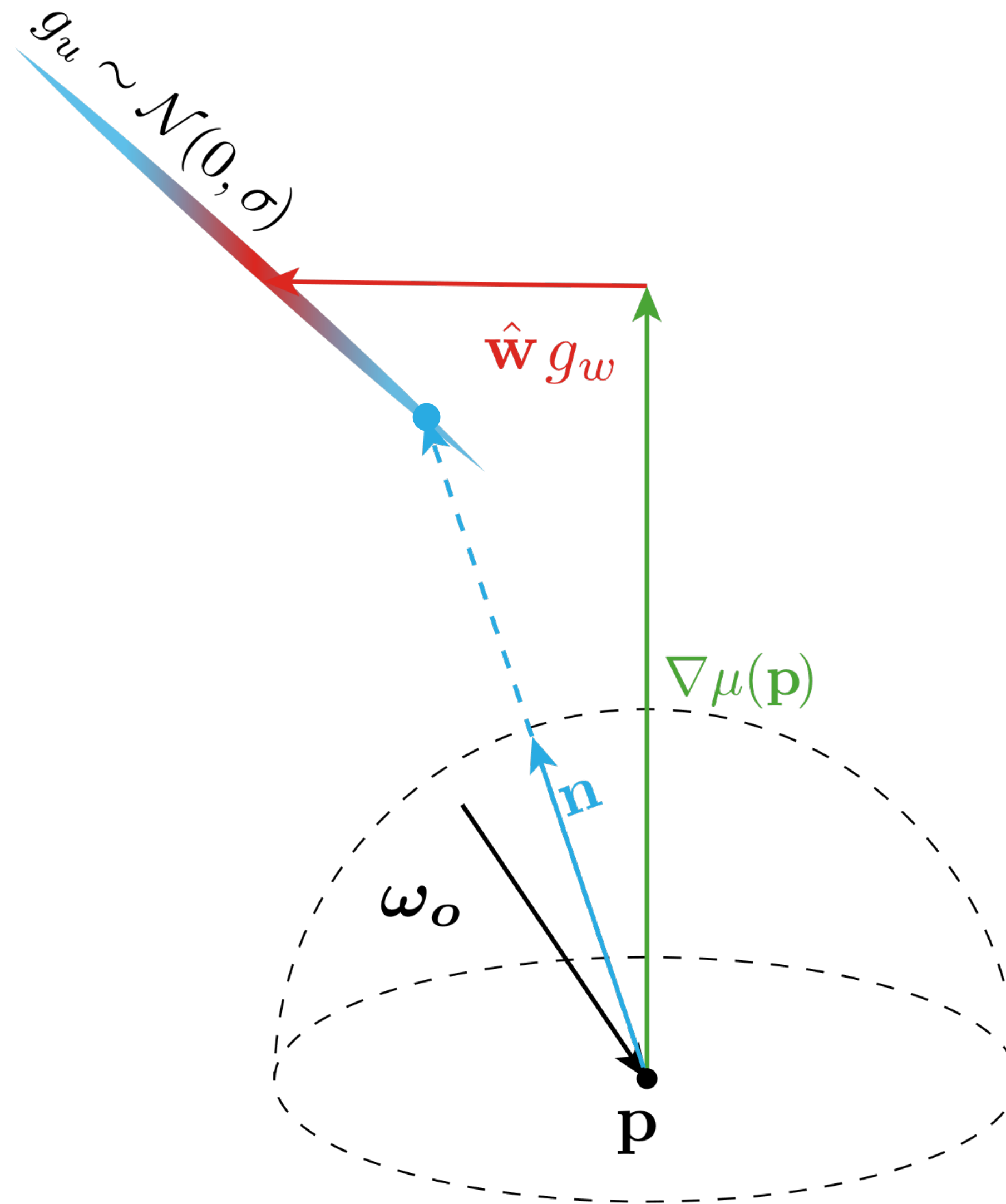
Normal Circle



Normal Circle

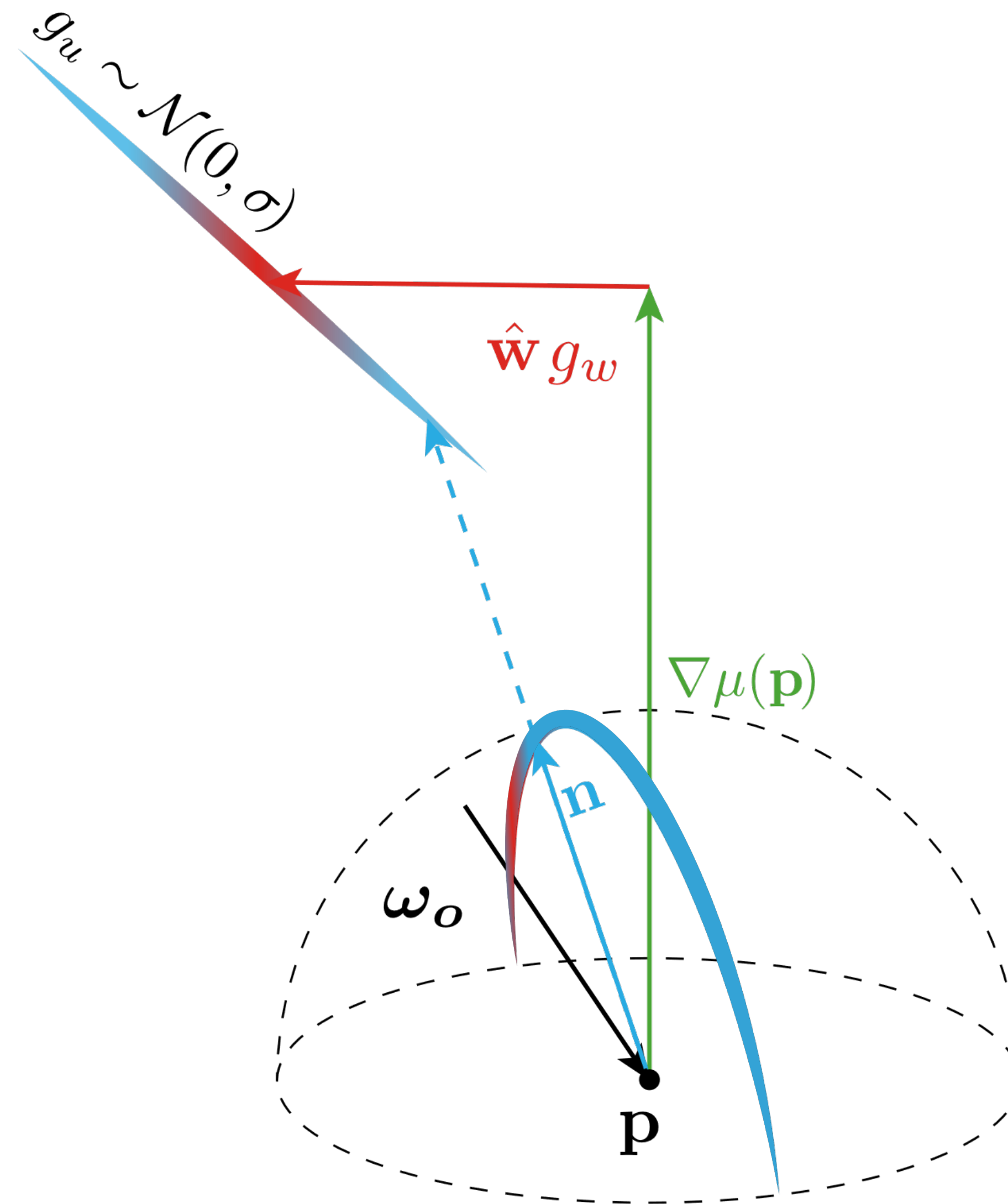


Normal Circle



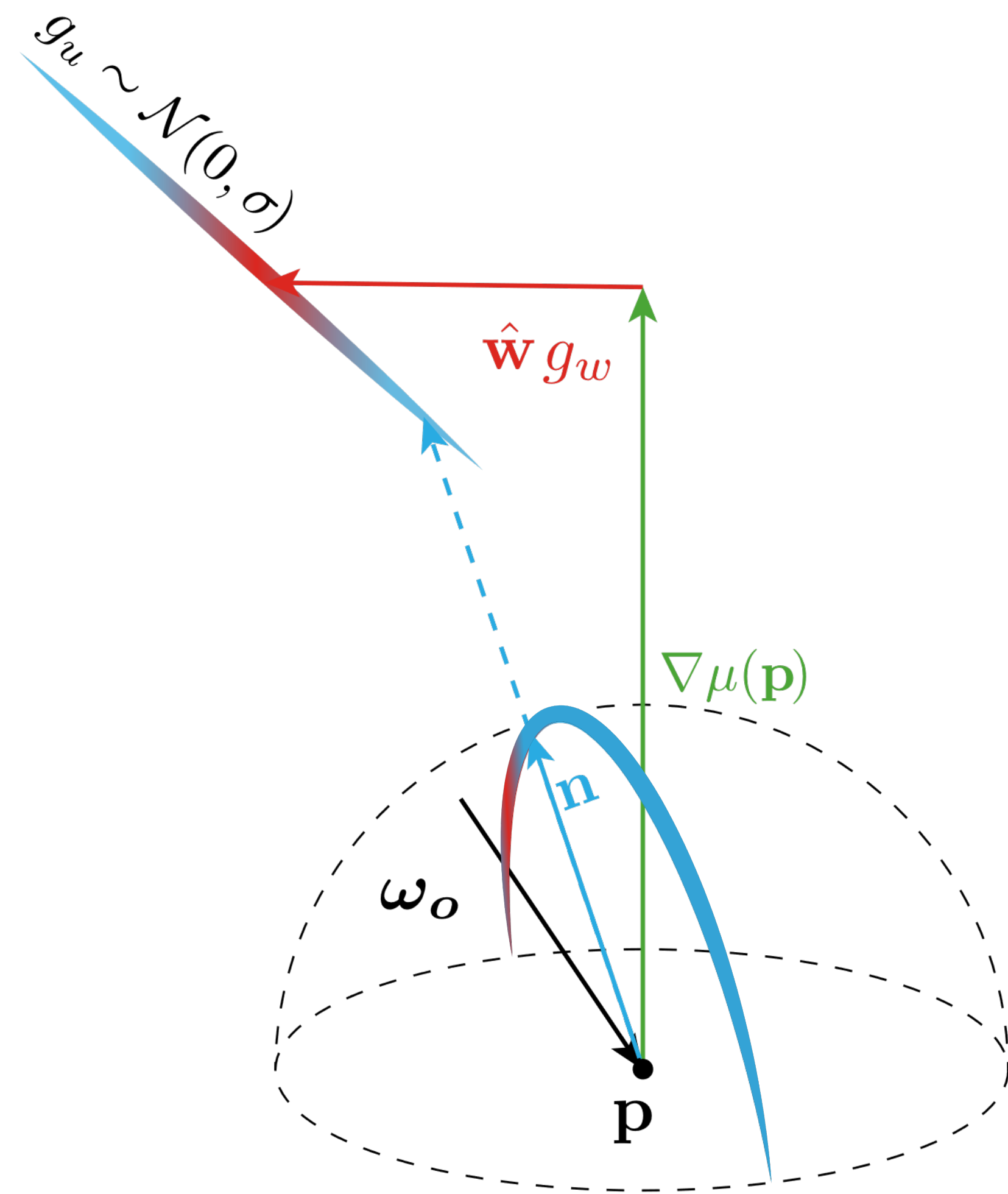
$$\mathbf{n} = \frac{\mathbf{g}}{\|\mathbf{g}\|}$$

Normal Circle

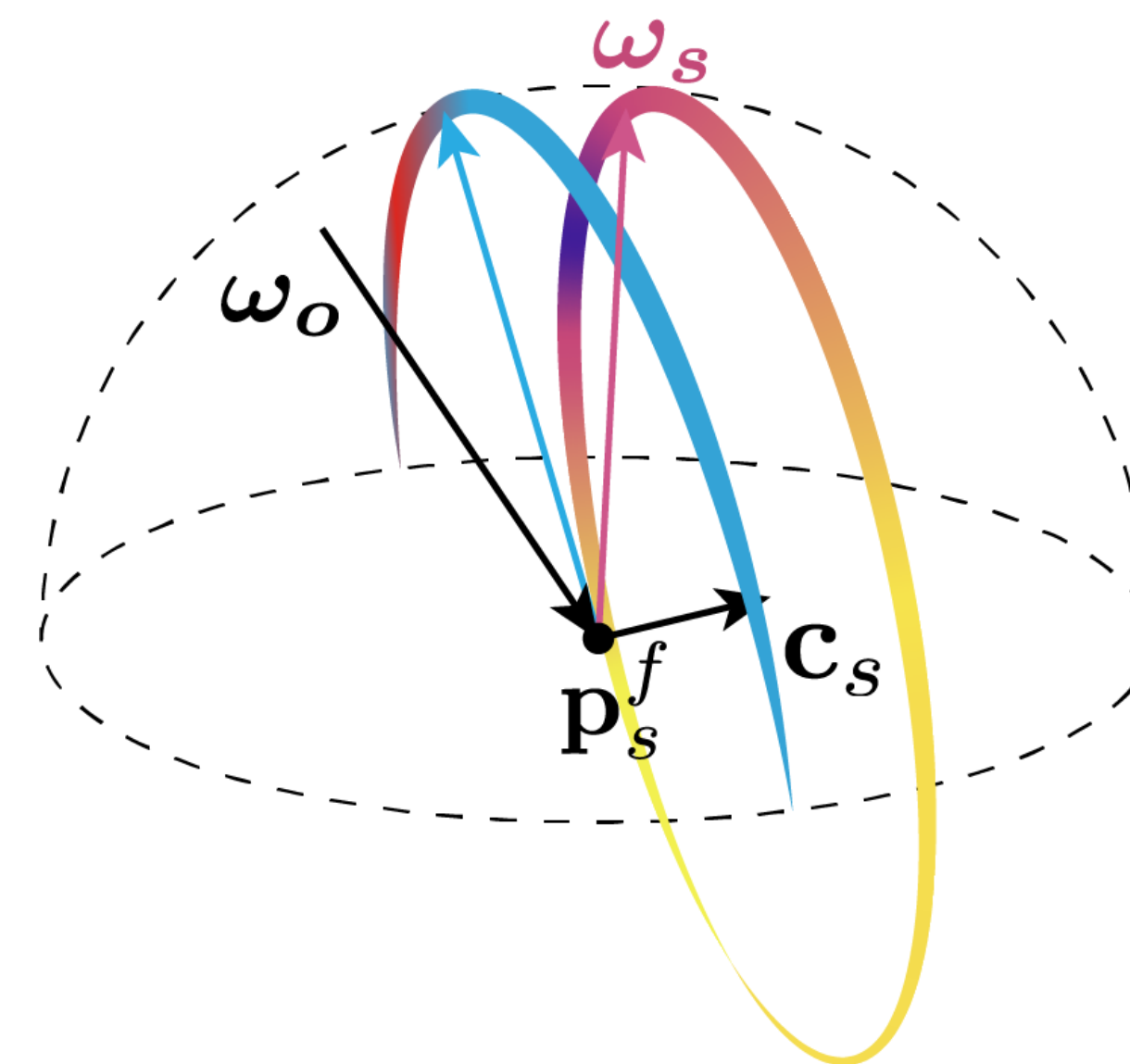


$$\mathbf{n} = \frac{\mathbf{g}}{\|\mathbf{g}\|}$$

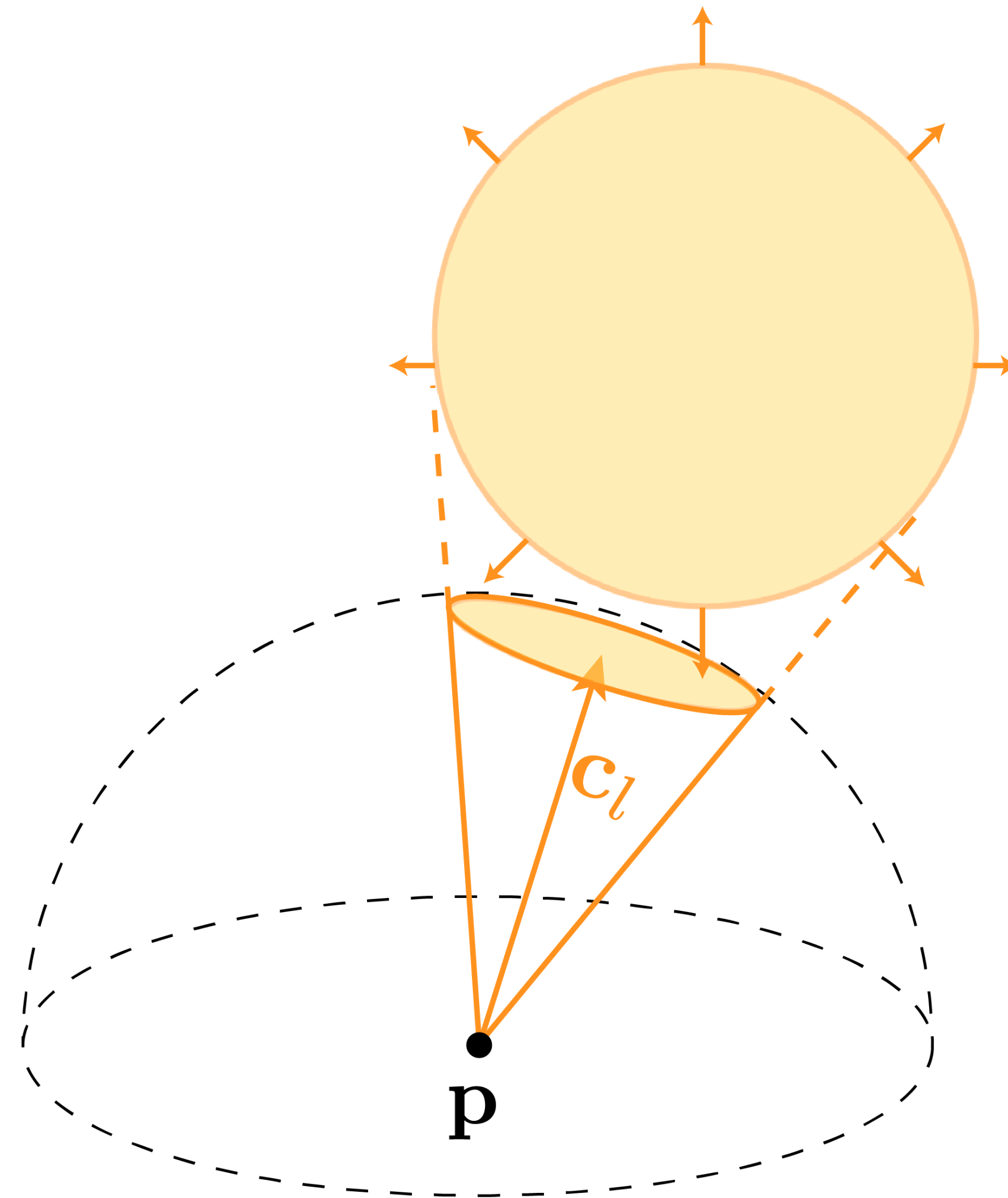
Scattering Circle



Reflect ω_o about n

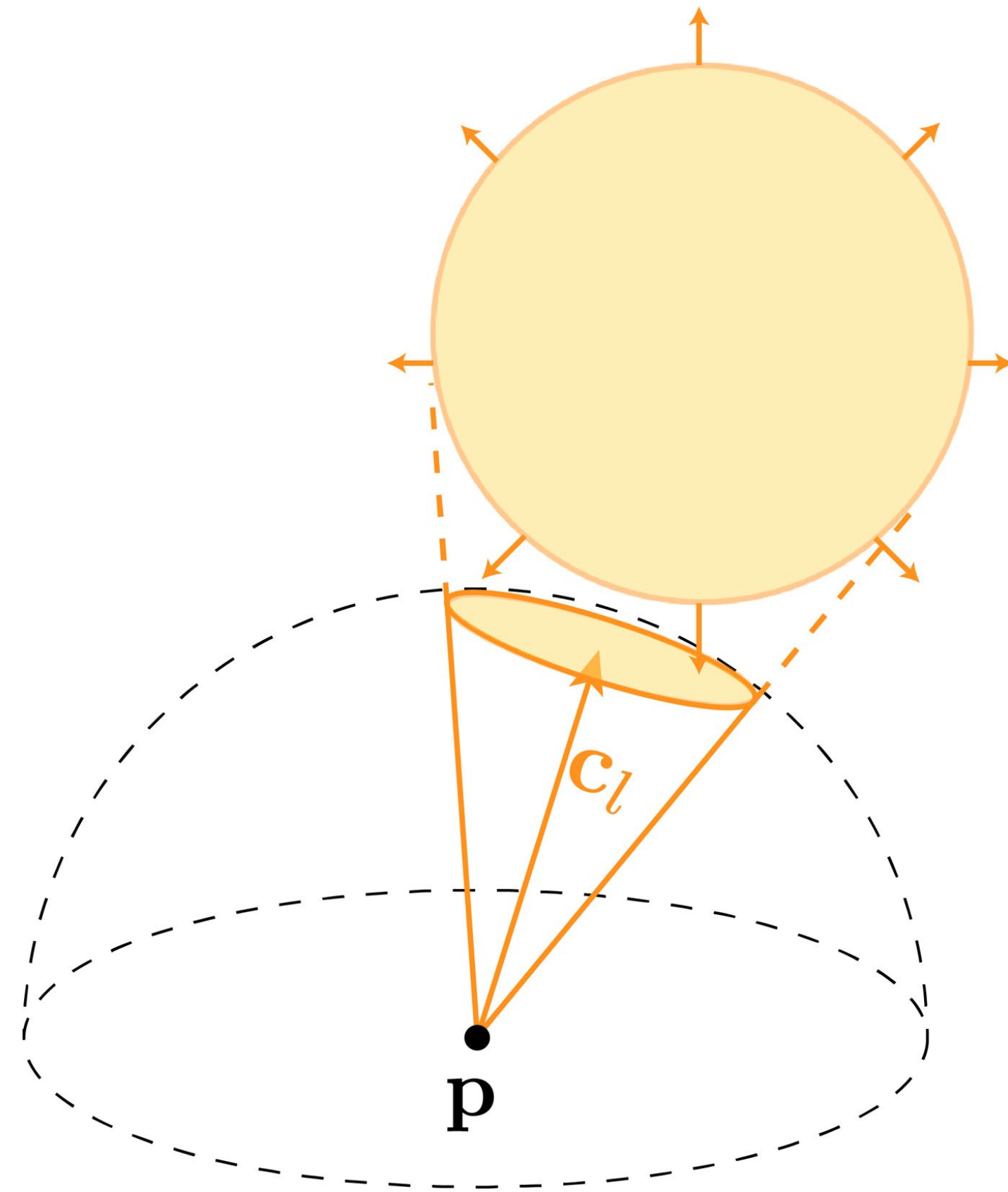


Light Circle Cap

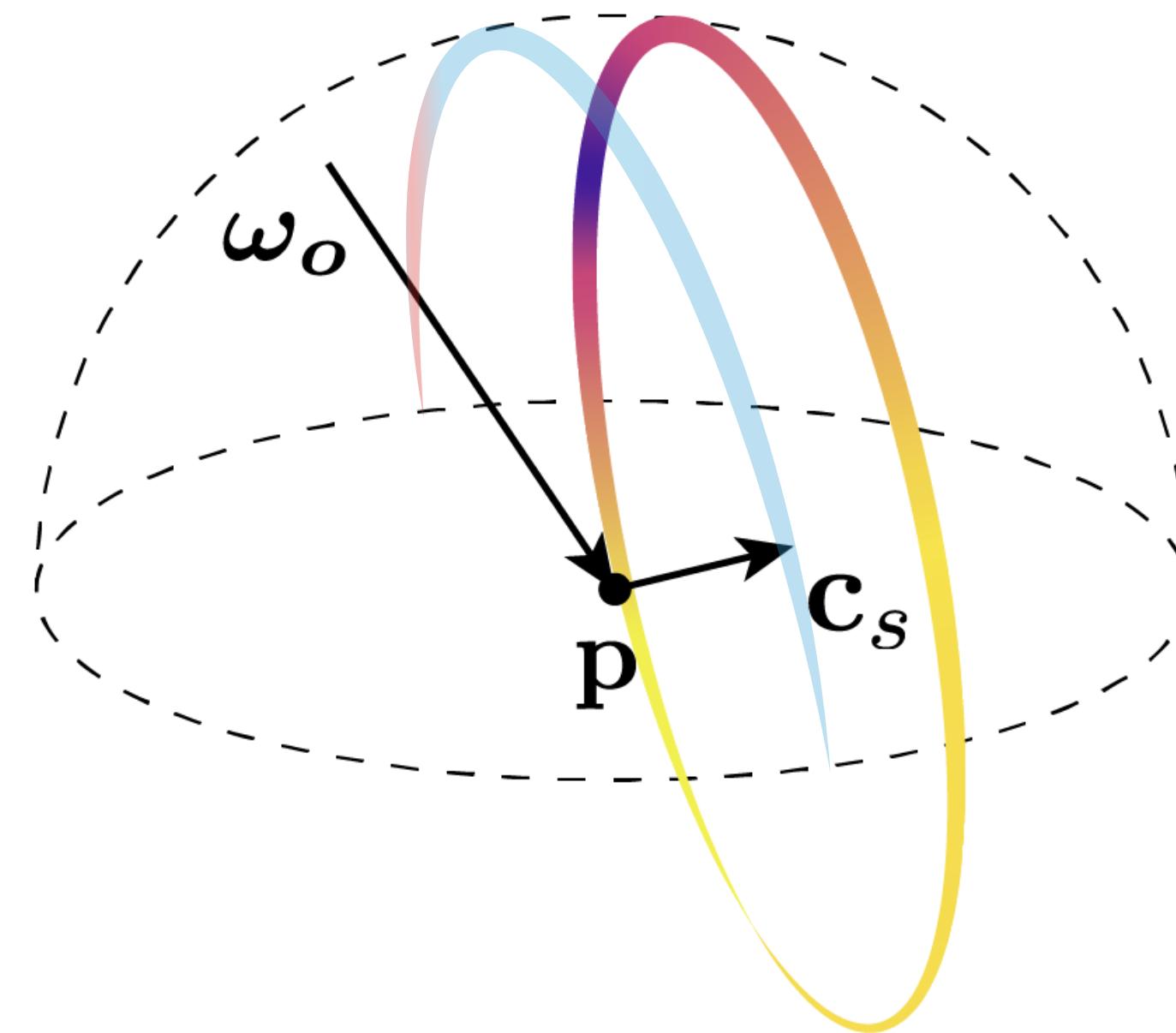


Double Circle Model

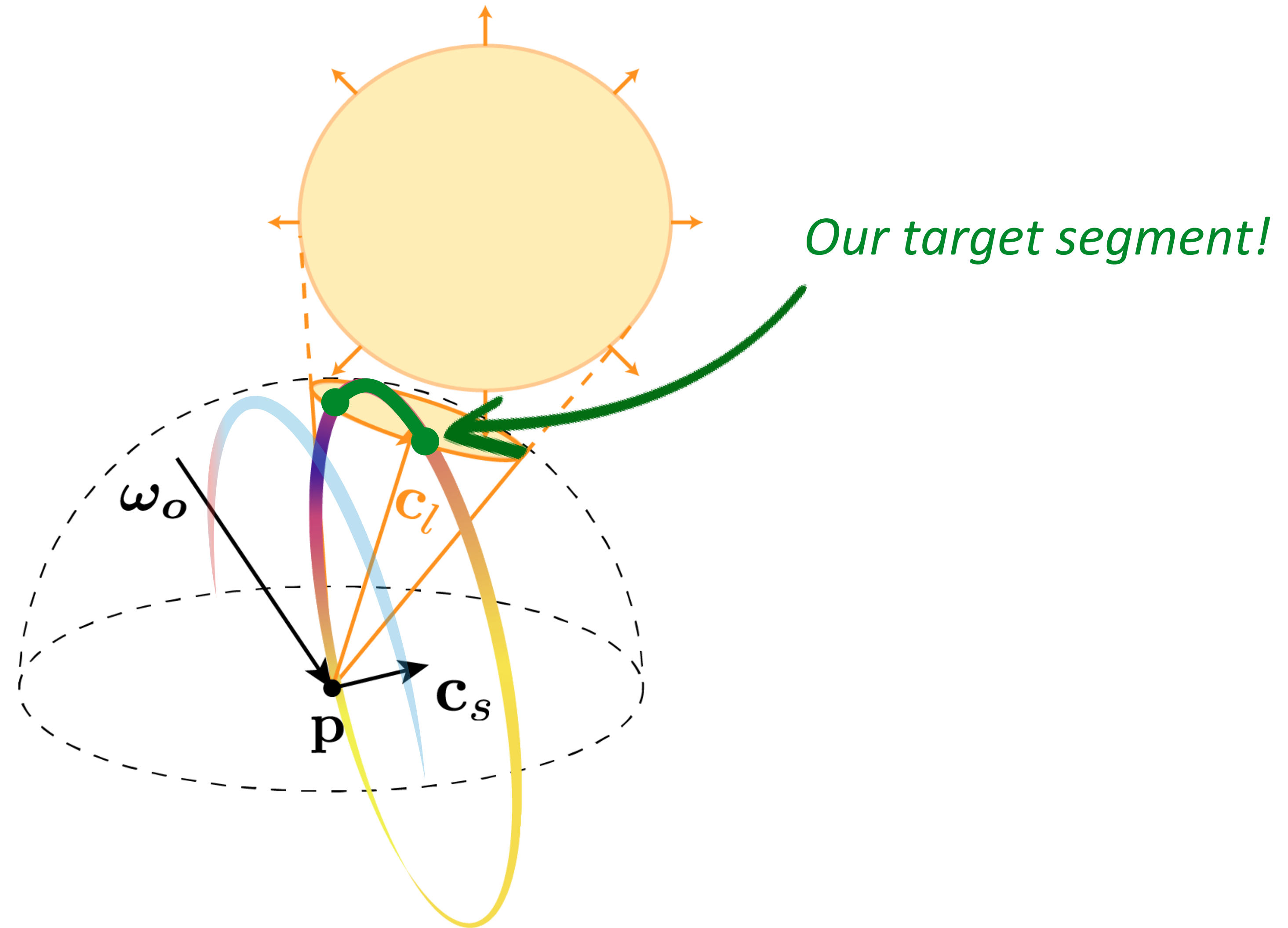
Light Circle Cap



Scattering Circle



Double Circle Model



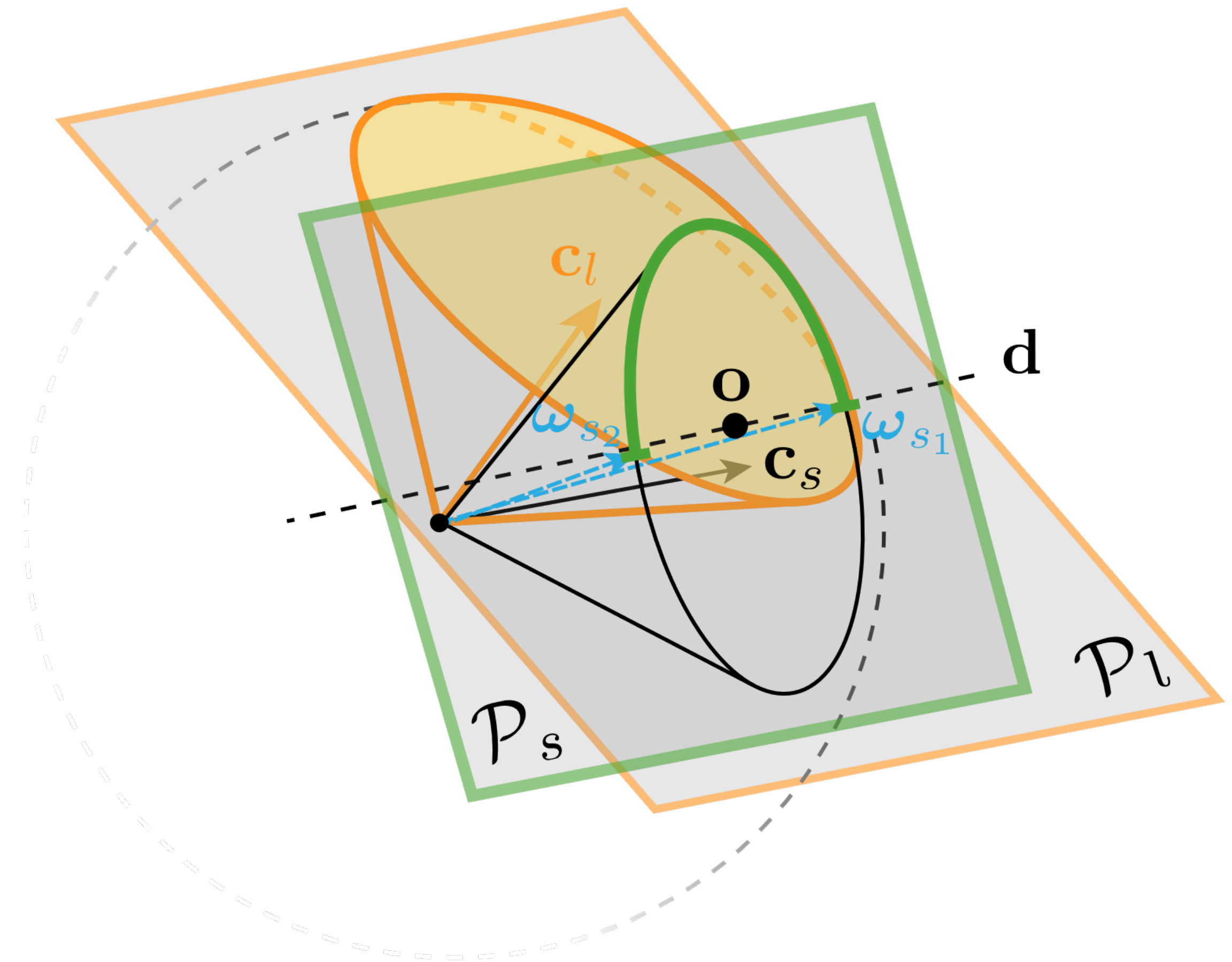
Double Circle Model

More Details in the paper

Simple to solve for the range of scattering directions!

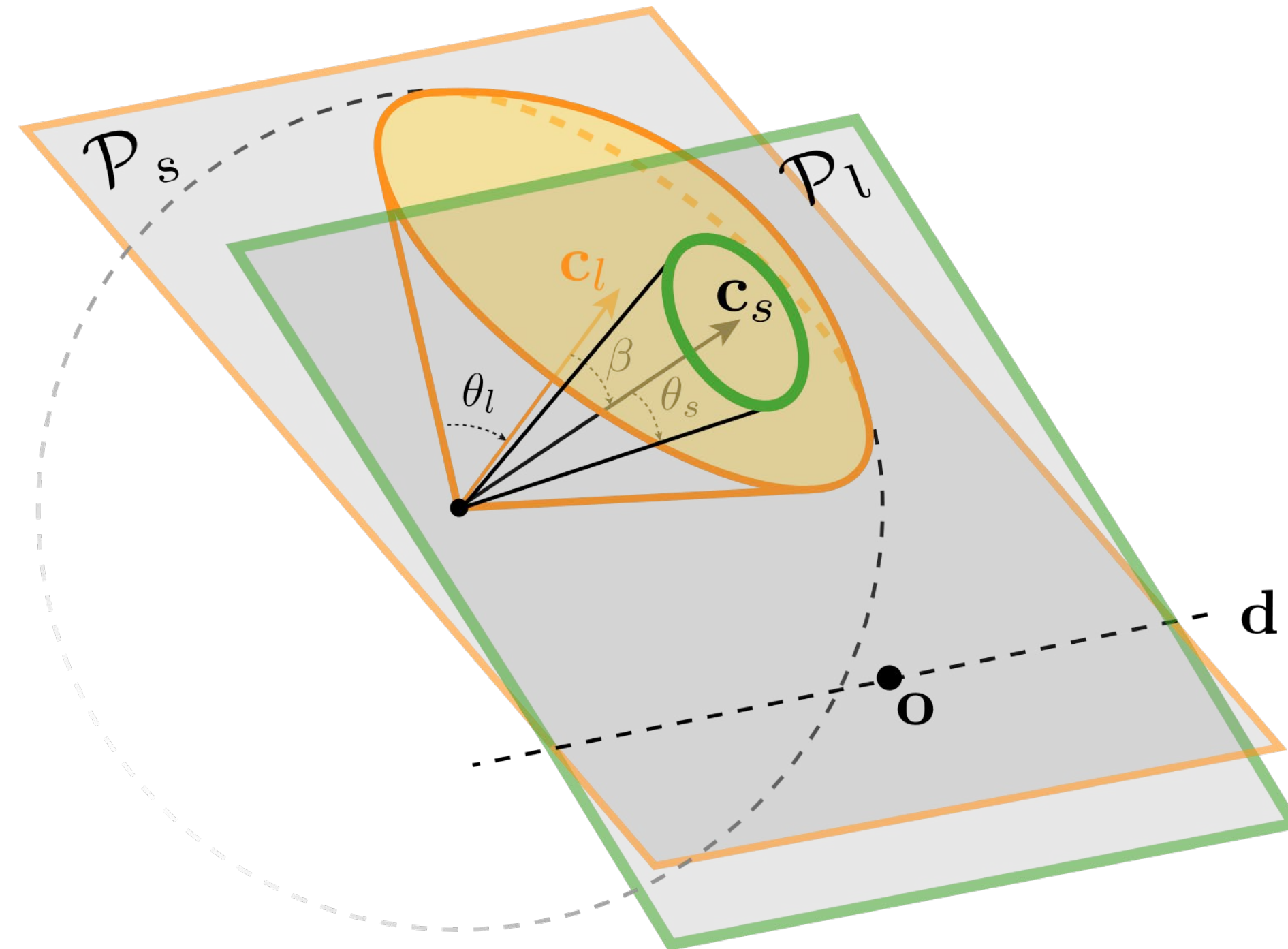
A 3x3 linear system

$$\begin{bmatrix} - & \mathbf{c}_s^\top & - \\ - & \mathbf{c}_l^\top & - \\ - & \mathbf{d}^\top & - \end{bmatrix} \mathbf{o} = \begin{bmatrix} \mathbf{c}_s \cdot \mathbf{c}_s \\ \mathbf{c}_l \cdot \mathbf{c}_l \\ 0 \end{bmatrix}$$

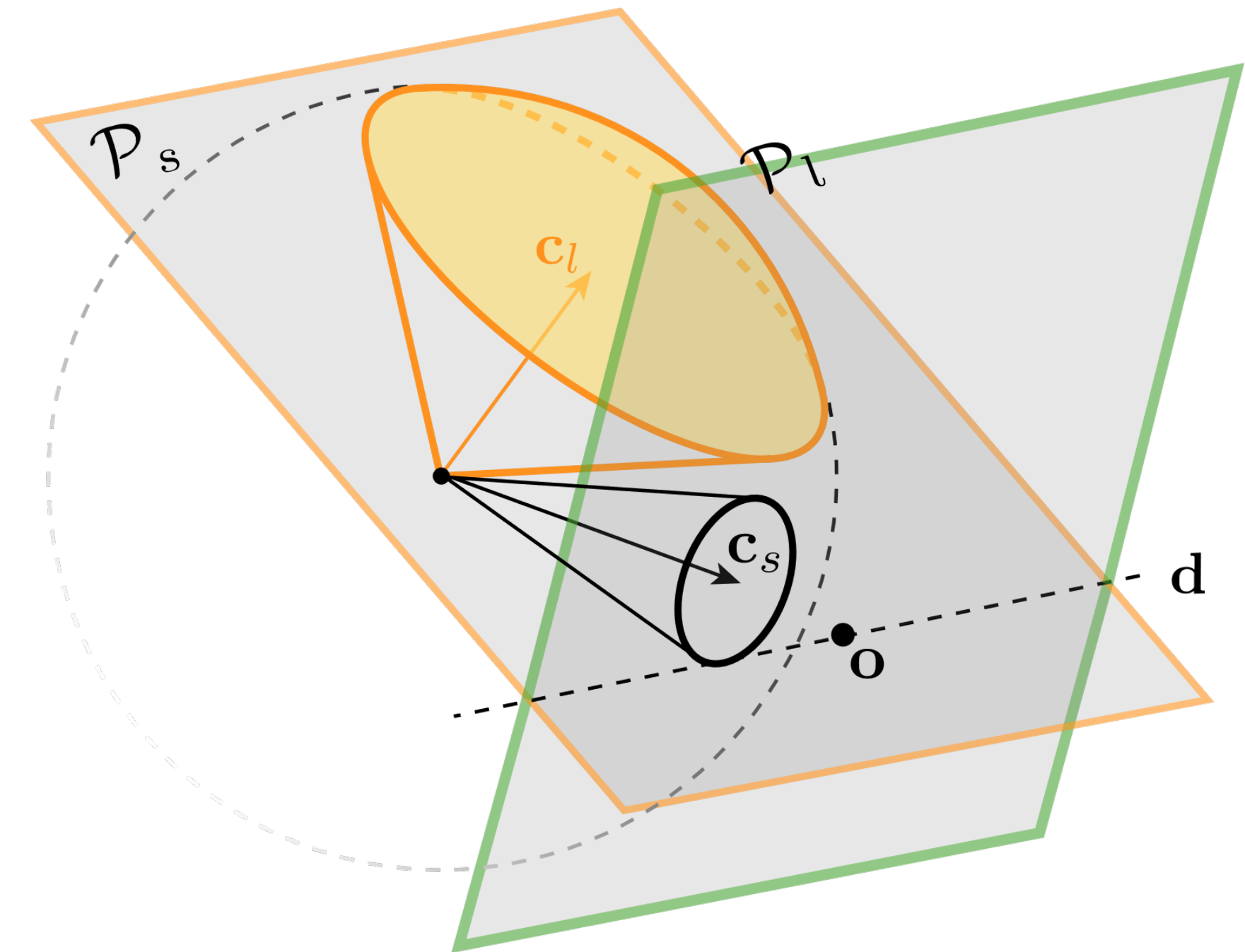


Double Circle Model – Other Cases

More Details in the paper



Fully contained



Missed

100

The diagram illustrates the geometric interpretation of the Bloch-embergen model for the Zeeman effect. It features a large yellow sphere representing the nucleus, with orange arrows indicating its magnetic field. A smaller orange sphere represents the electron, with green arrows indicating its magnetic field. The diagram shows the relative orientations of the nuclear and electron magnetic fields, labeled with symbols like ω_0 , ω_{s1} , ω_{s2} , and $\mathbf{c}_s$, and a point p .

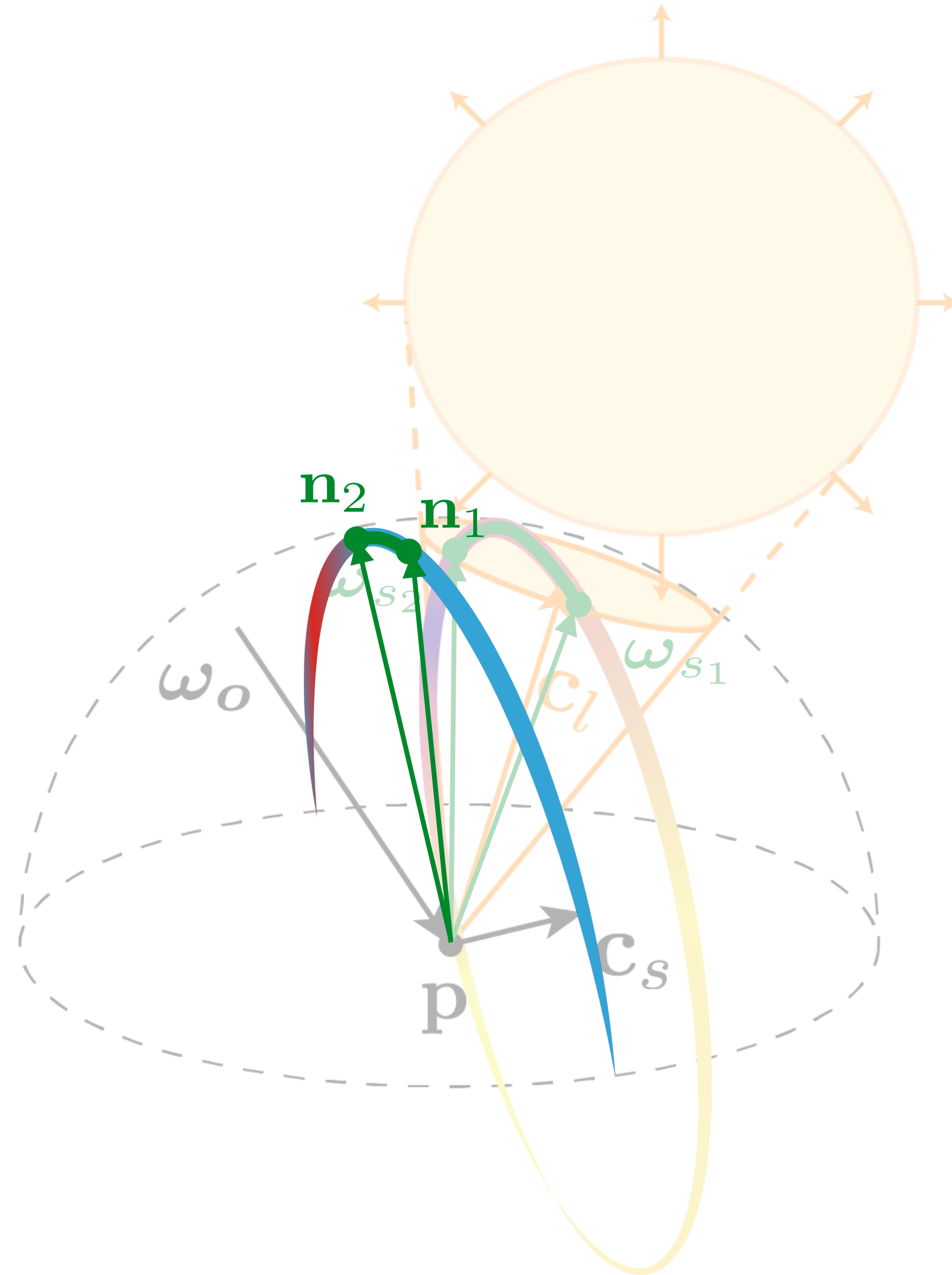
Restricted sampling domain

Scattering direction range

$[\omega_{s_1} , \omega_{s_2}]$

Normal vector range

$[\mathbf{n}_1 , \mathbf{n}_2]$



Restricted sampling domain

Scattering direction range

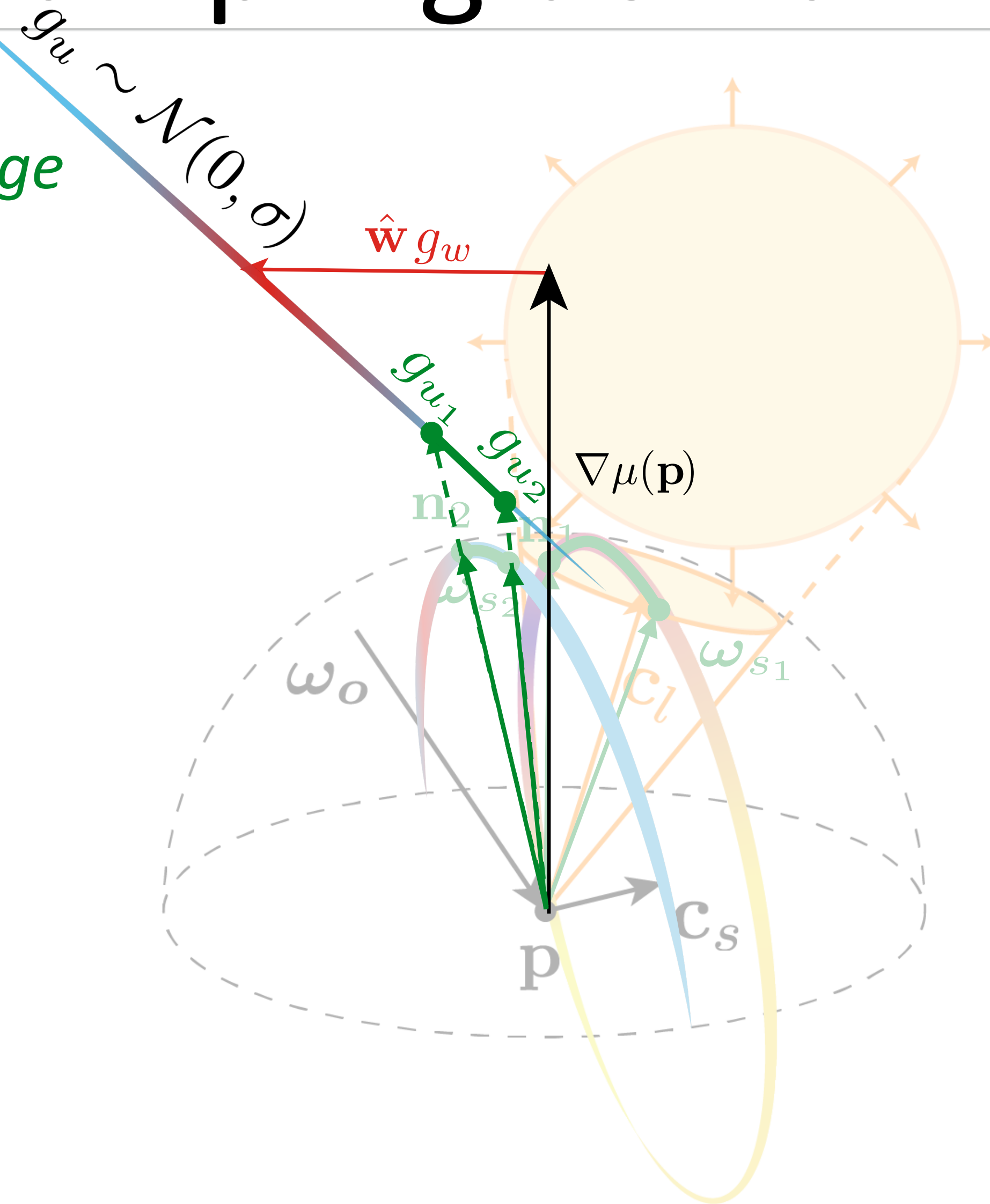
$[\omega_{s_1}, \omega_{s_2}]$

Normal vector range

$[\mathbf{n}_1, \mathbf{n}_2]$

Gradient range

$[g_{u_1}, g_{u_2}]$



Sample truncated Gaussian

Scattering direction range

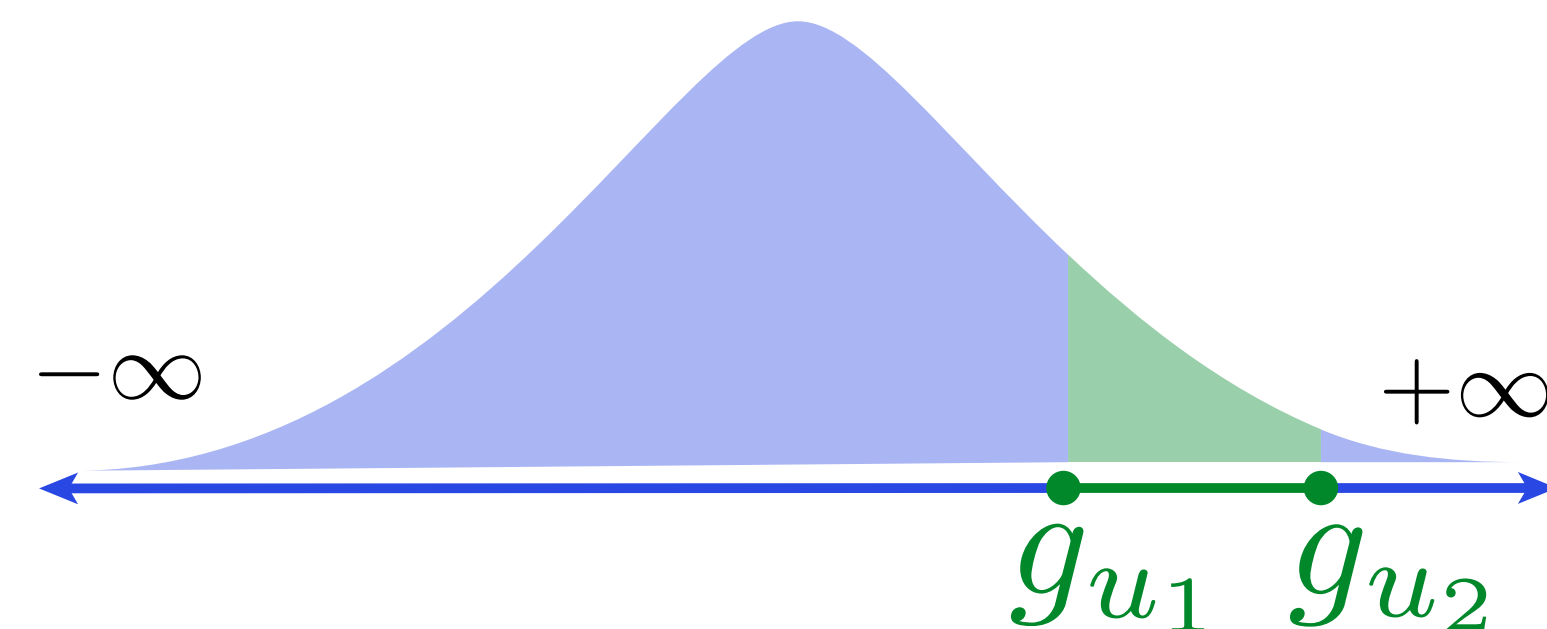
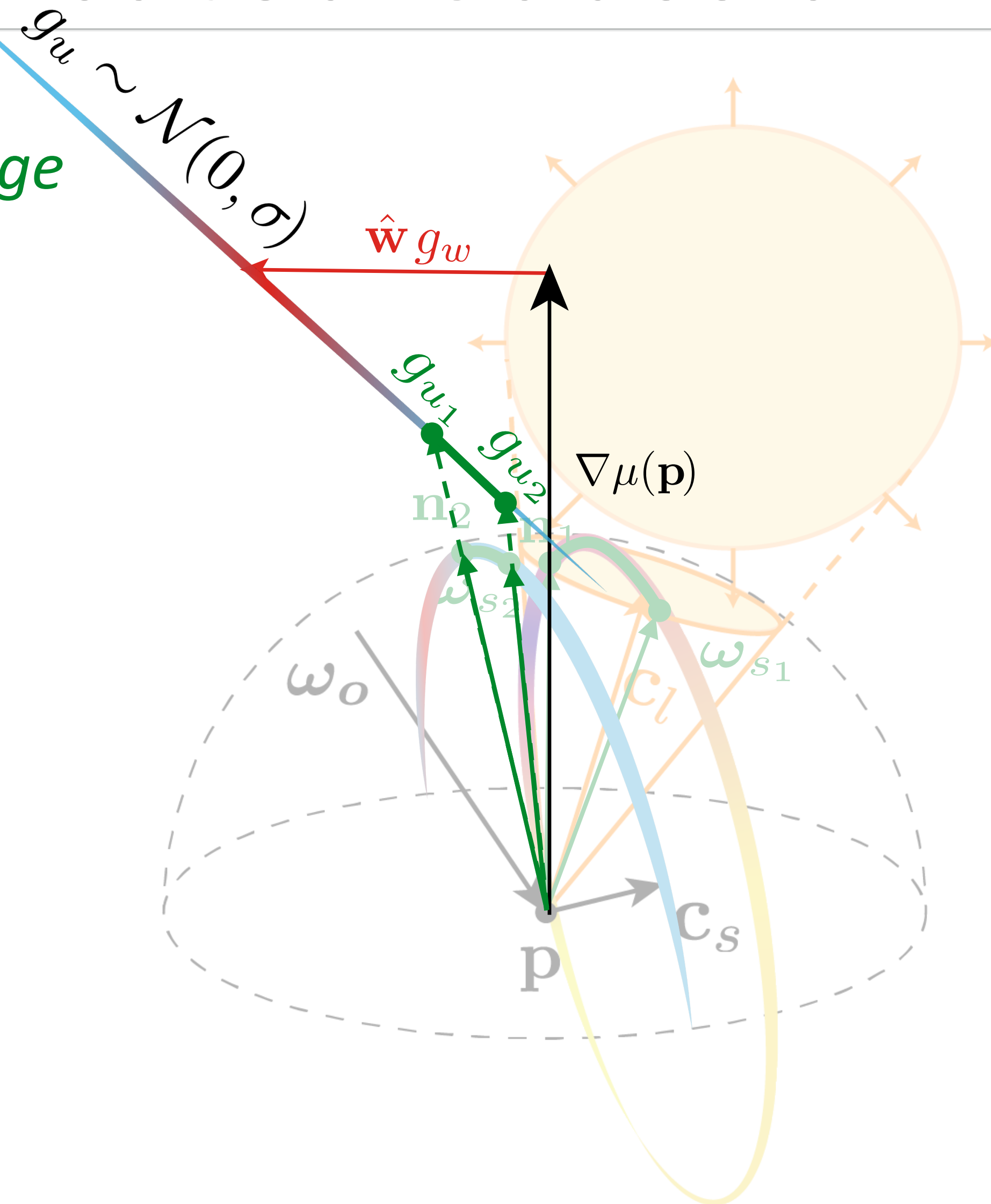
$[\omega_{s_1}, \omega_{s_2}]$

Normal vector range

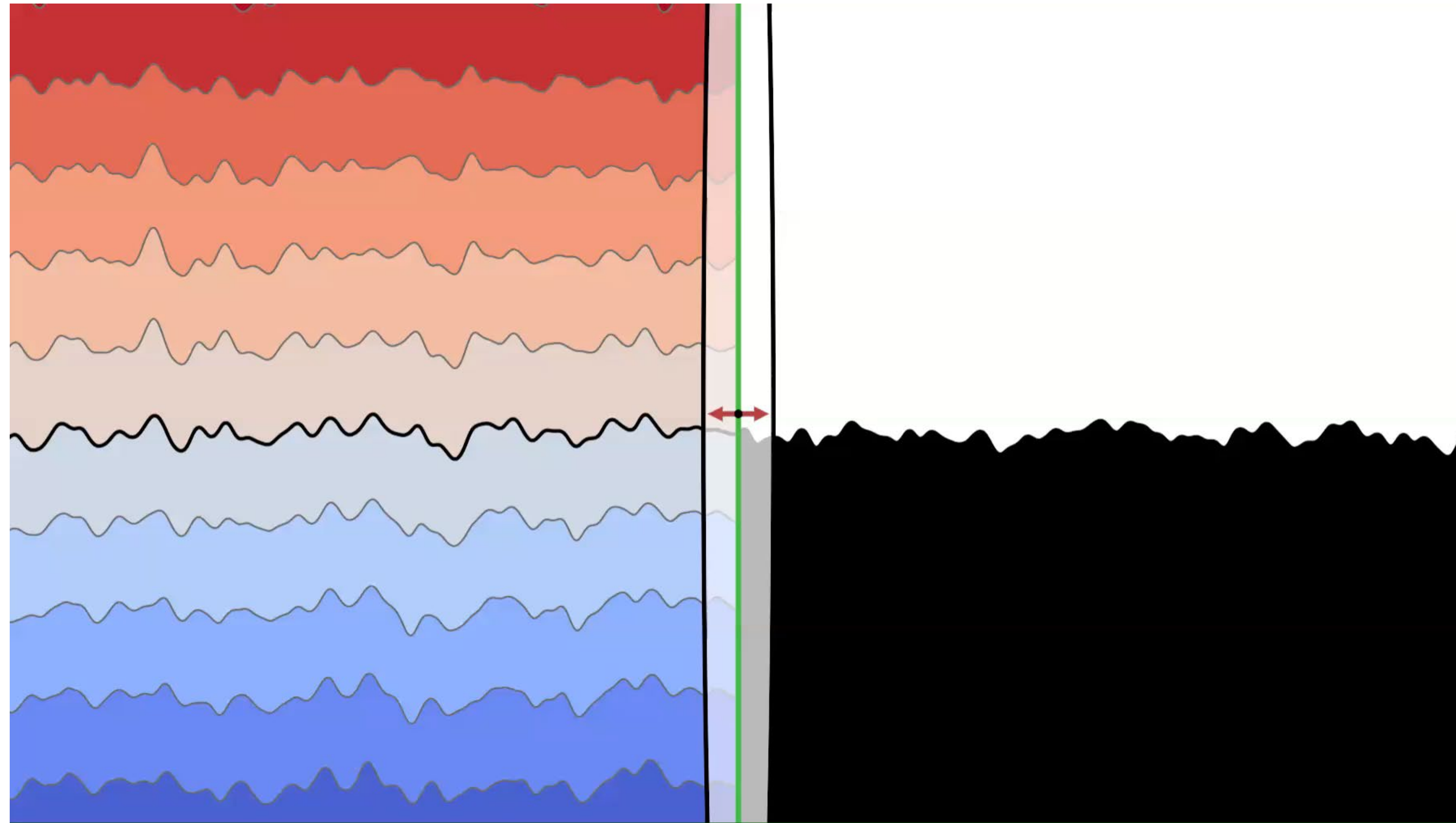
$[\mathbf{n}_1, \mathbf{n}_2]$

Gradient range

$[g_{u_1}, g_{u_2}]$



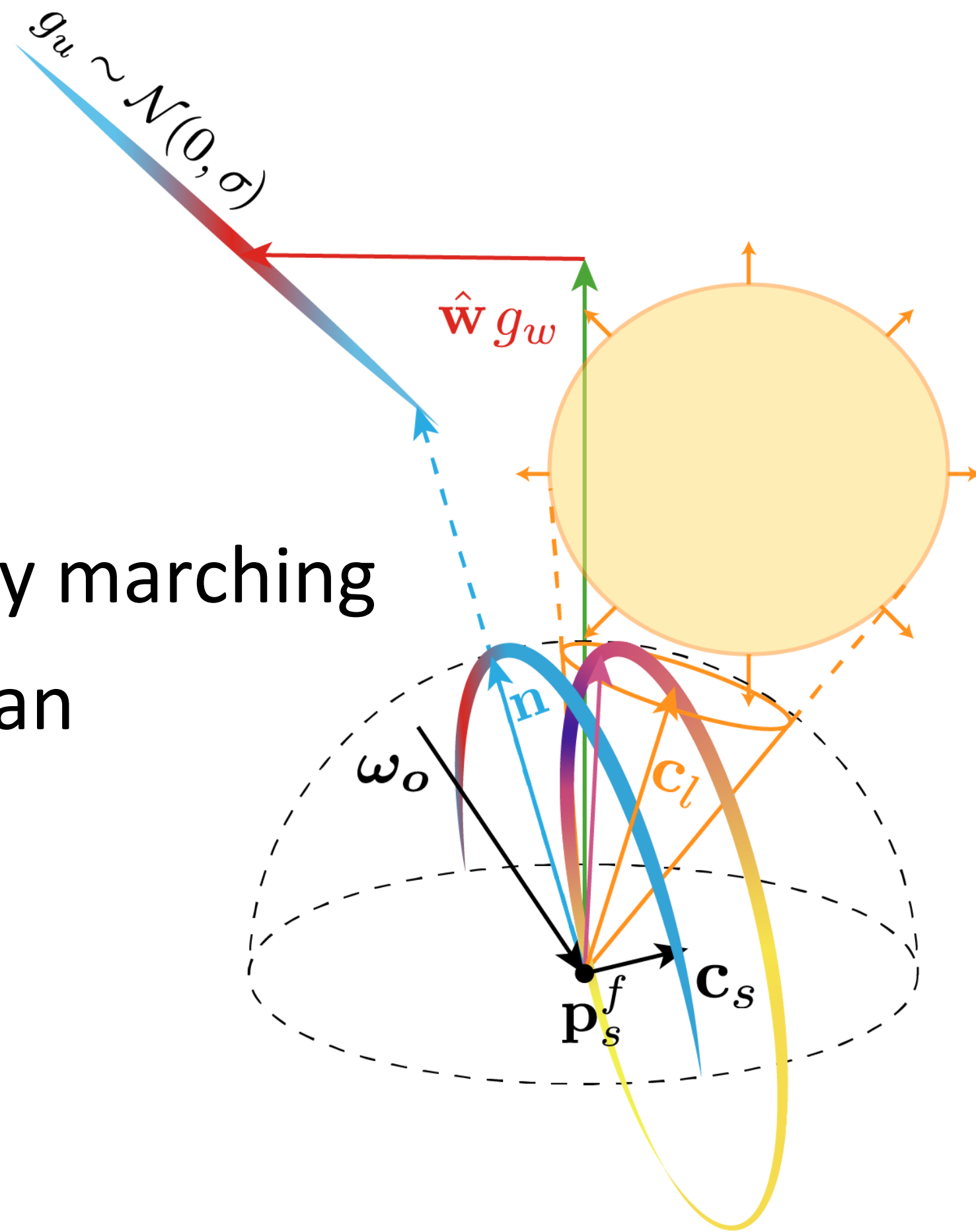
From Heightfield to General GPIS



From Heightfield to General GPIS

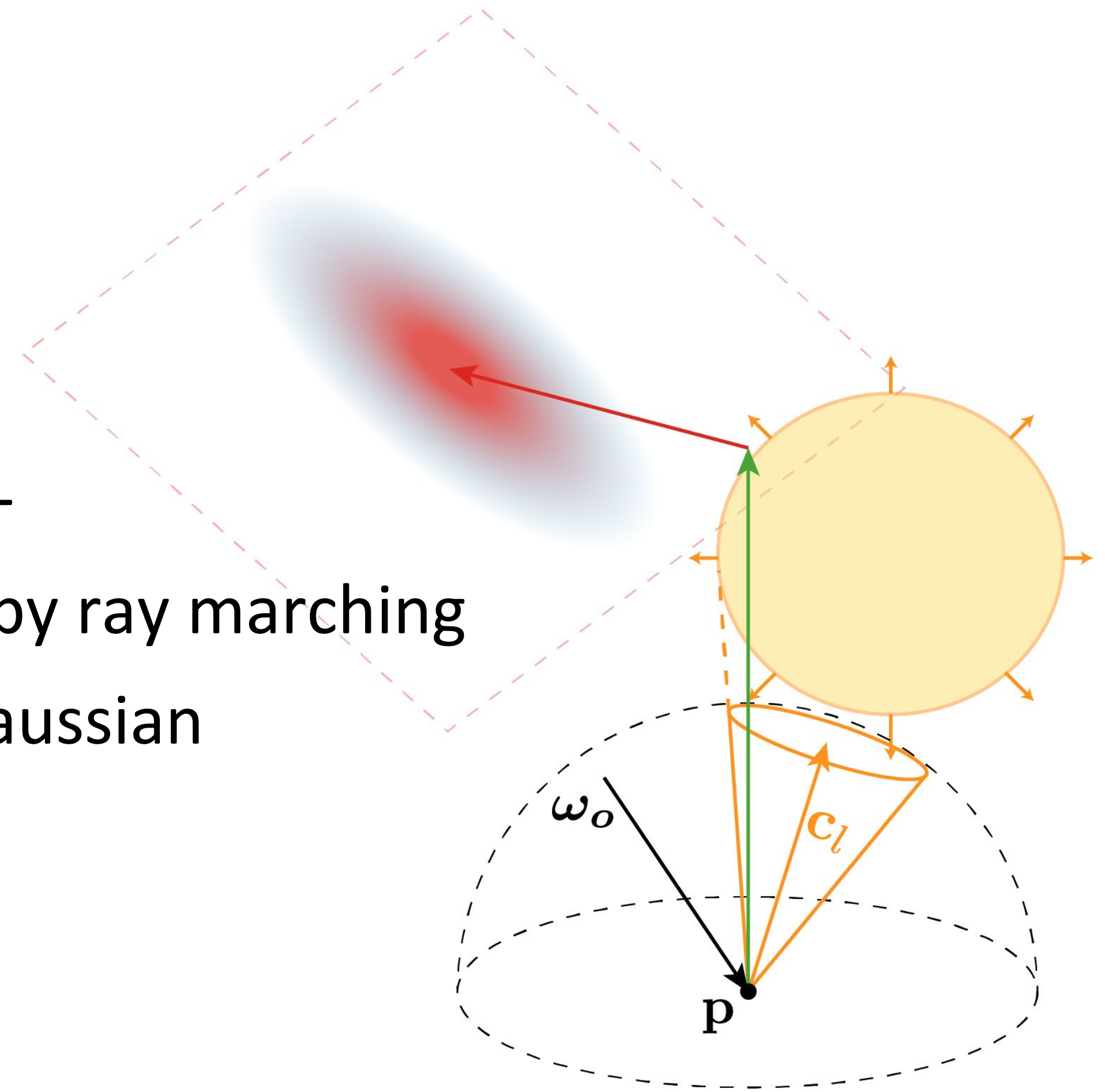
2 DoF –

1 fixed by ray marching
= **1D** Gaussian



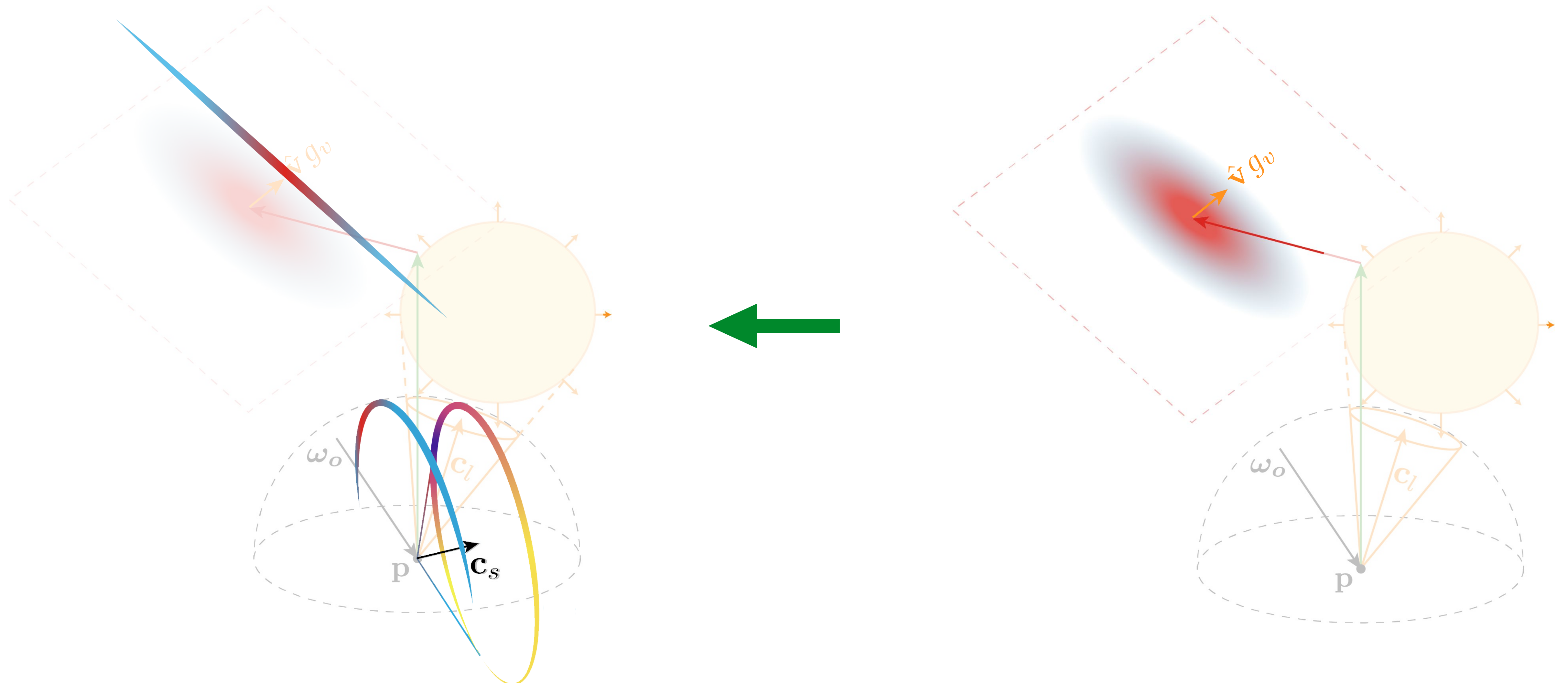
3 DoF –

1 fixed by ray marching
= **2D** Gaussian



From Heightfield to General GPIS

More Details in the paper





Results

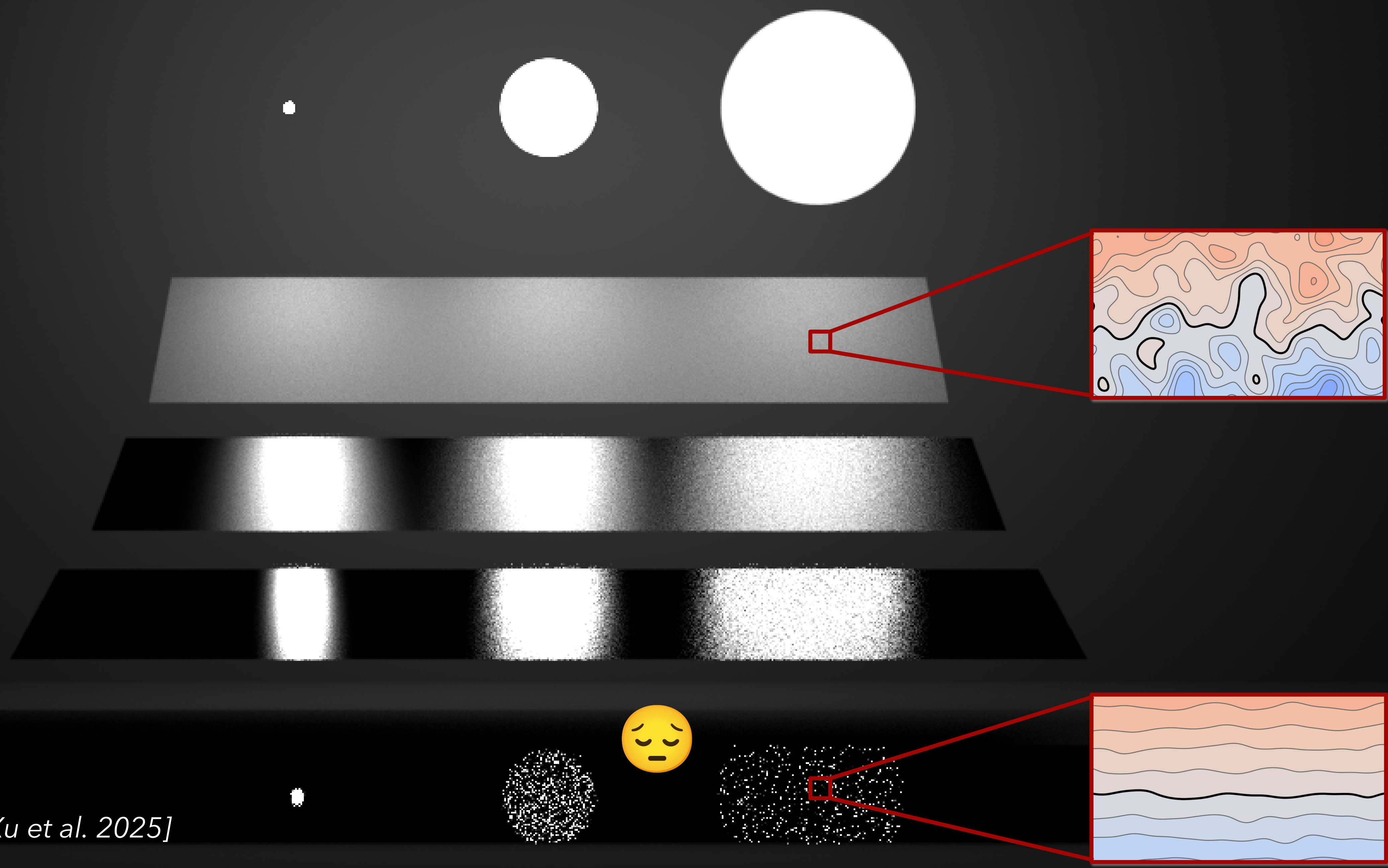


MSE:1.165

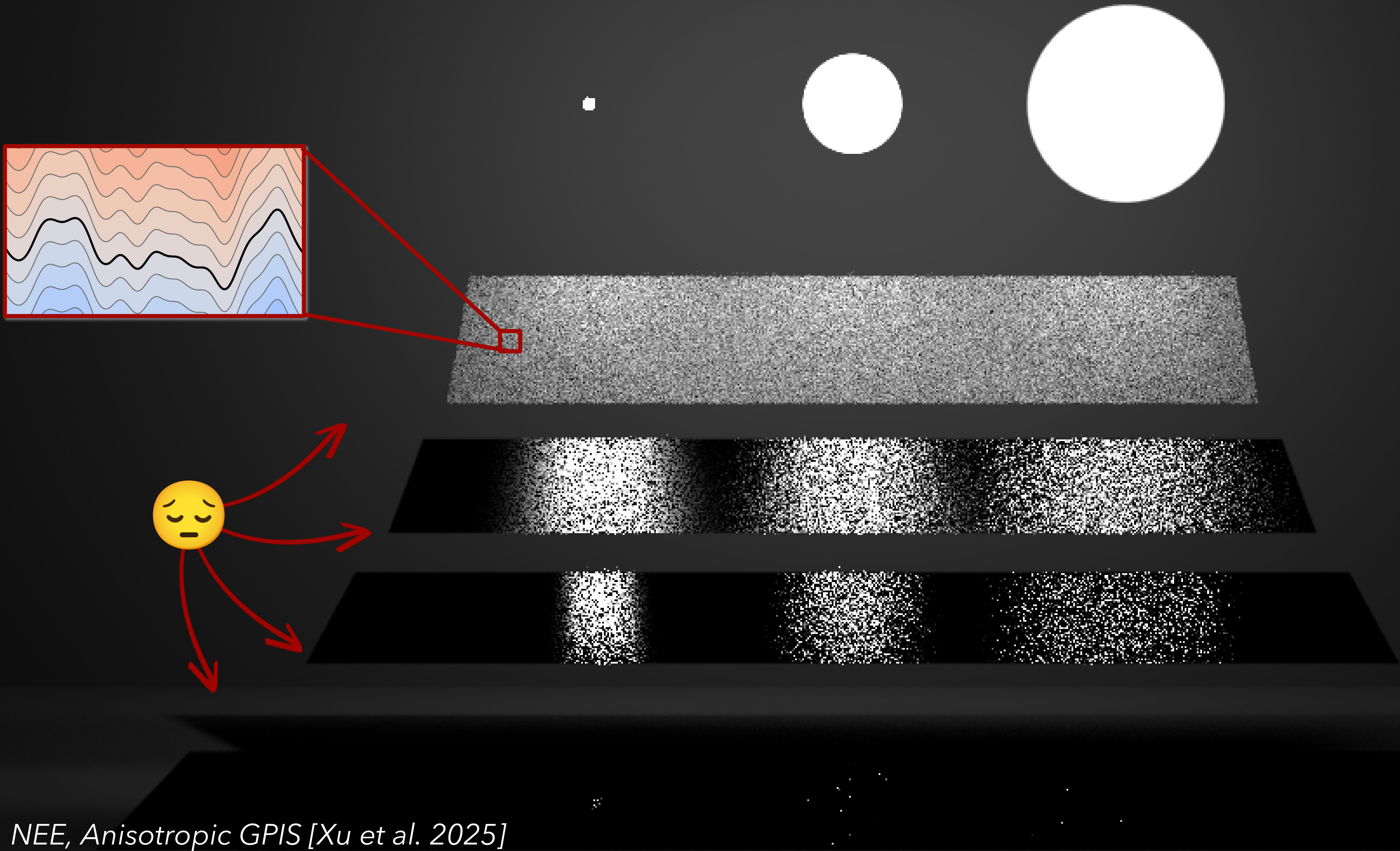


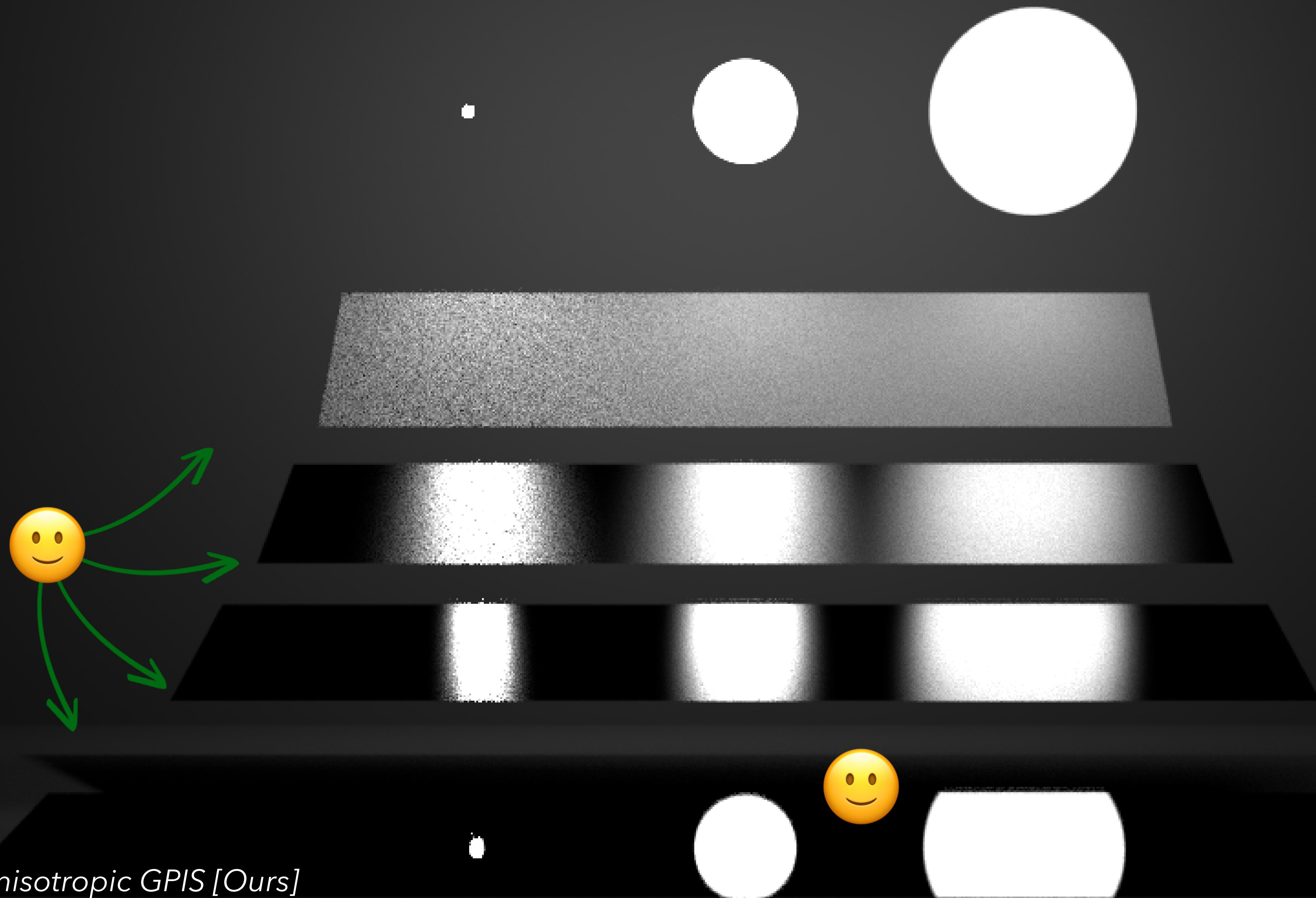
NEE, Anisotropic GPIS [Xu et al. 2025]





NEE, Isotropic GPIS [Xu et al. 2025]

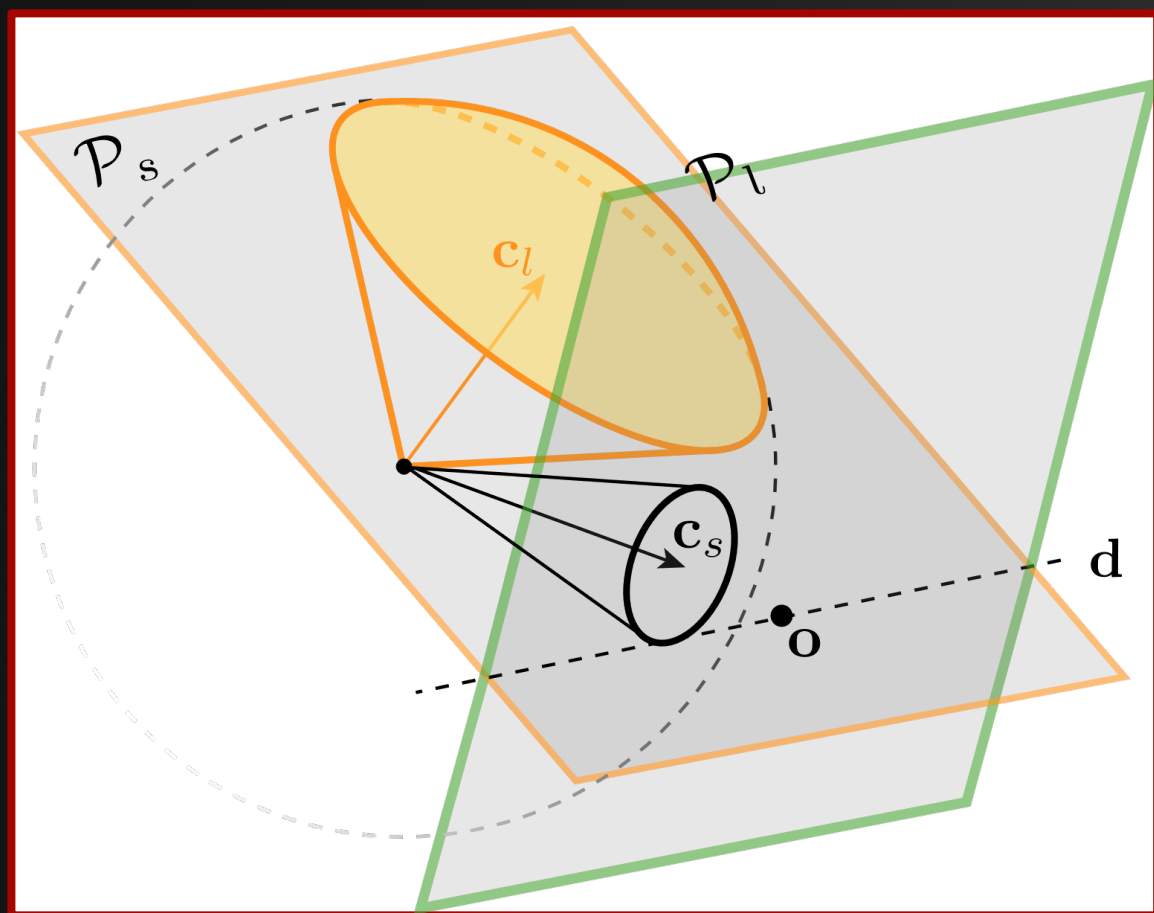




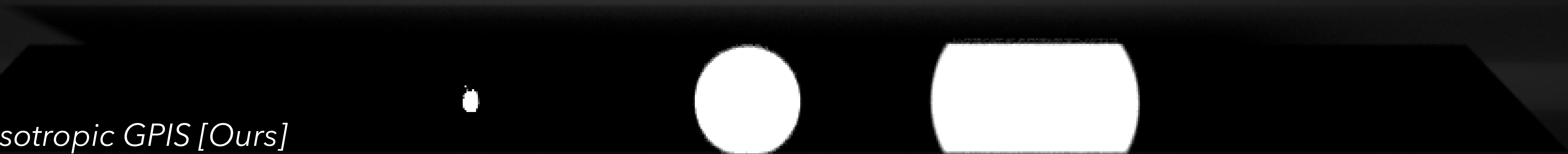
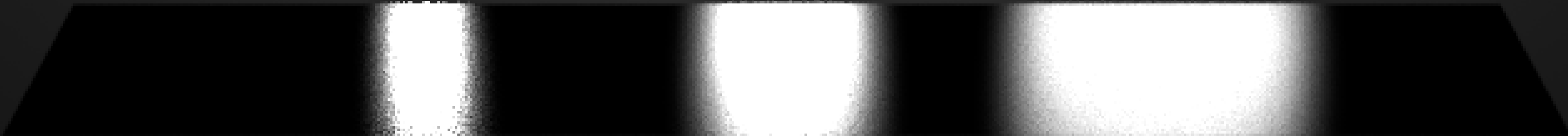
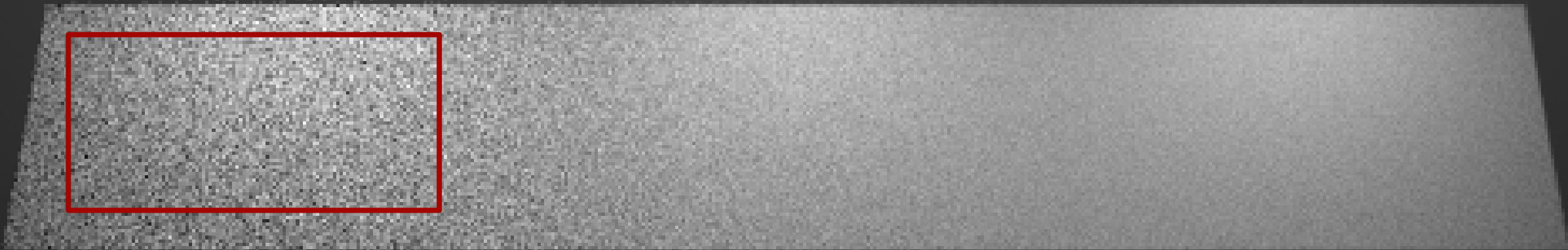
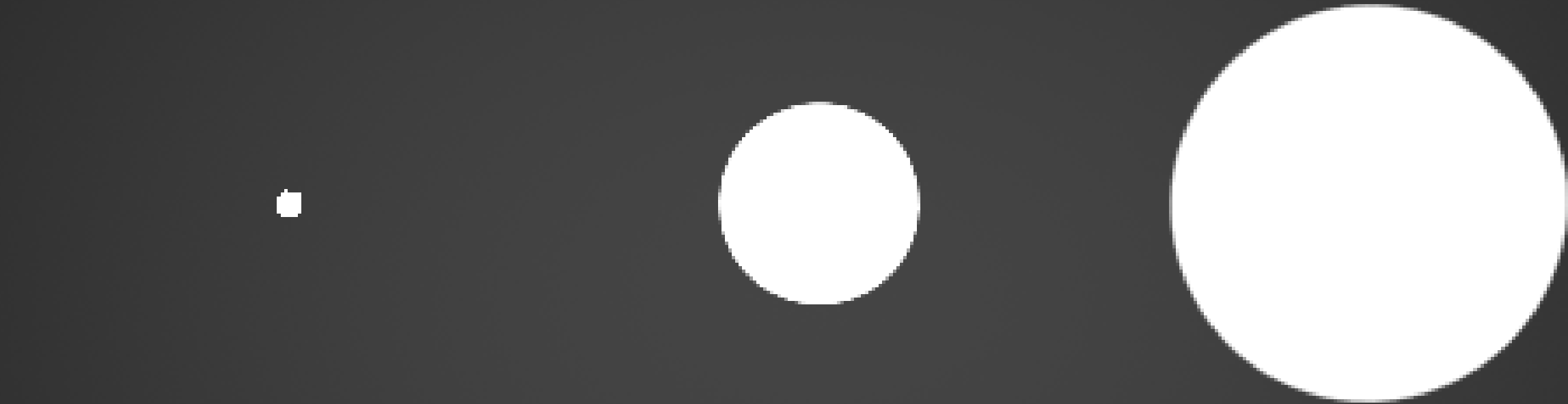
CPNEE, Anisotropic GPIS [Ours]



Limitation & Future Work

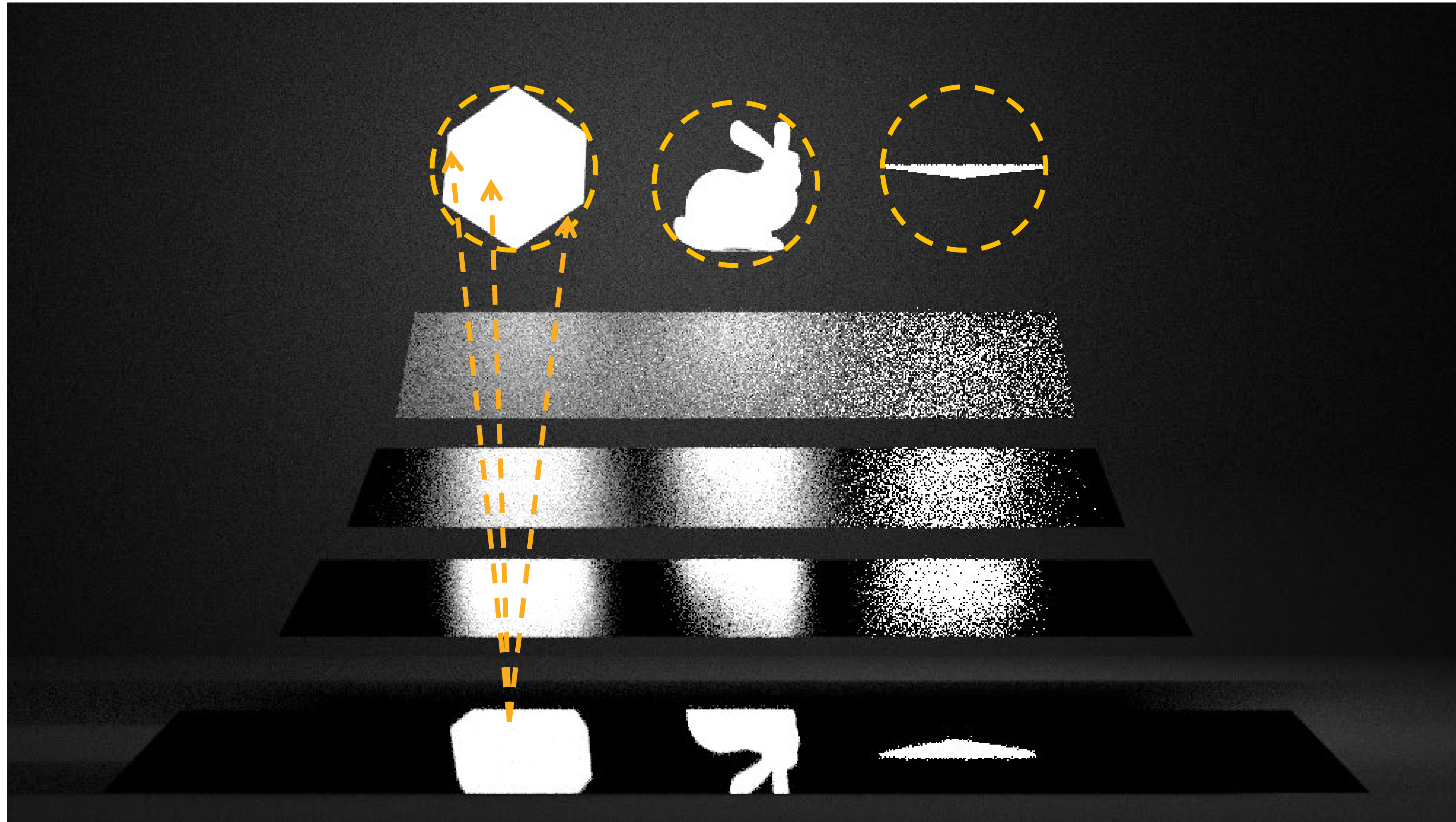


Missed intersection

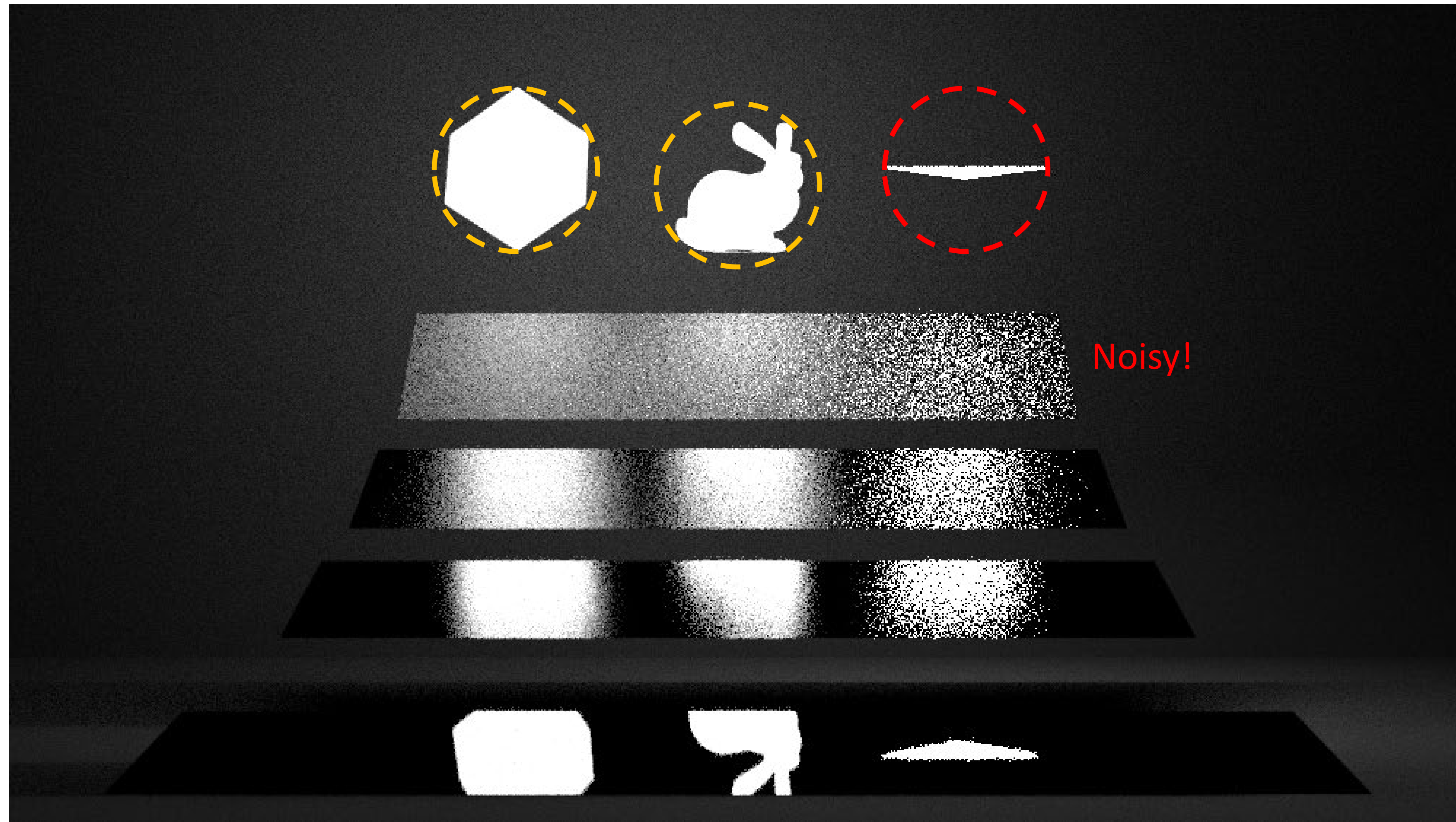


CPNEE, Anisotropic GPIS [Ours]

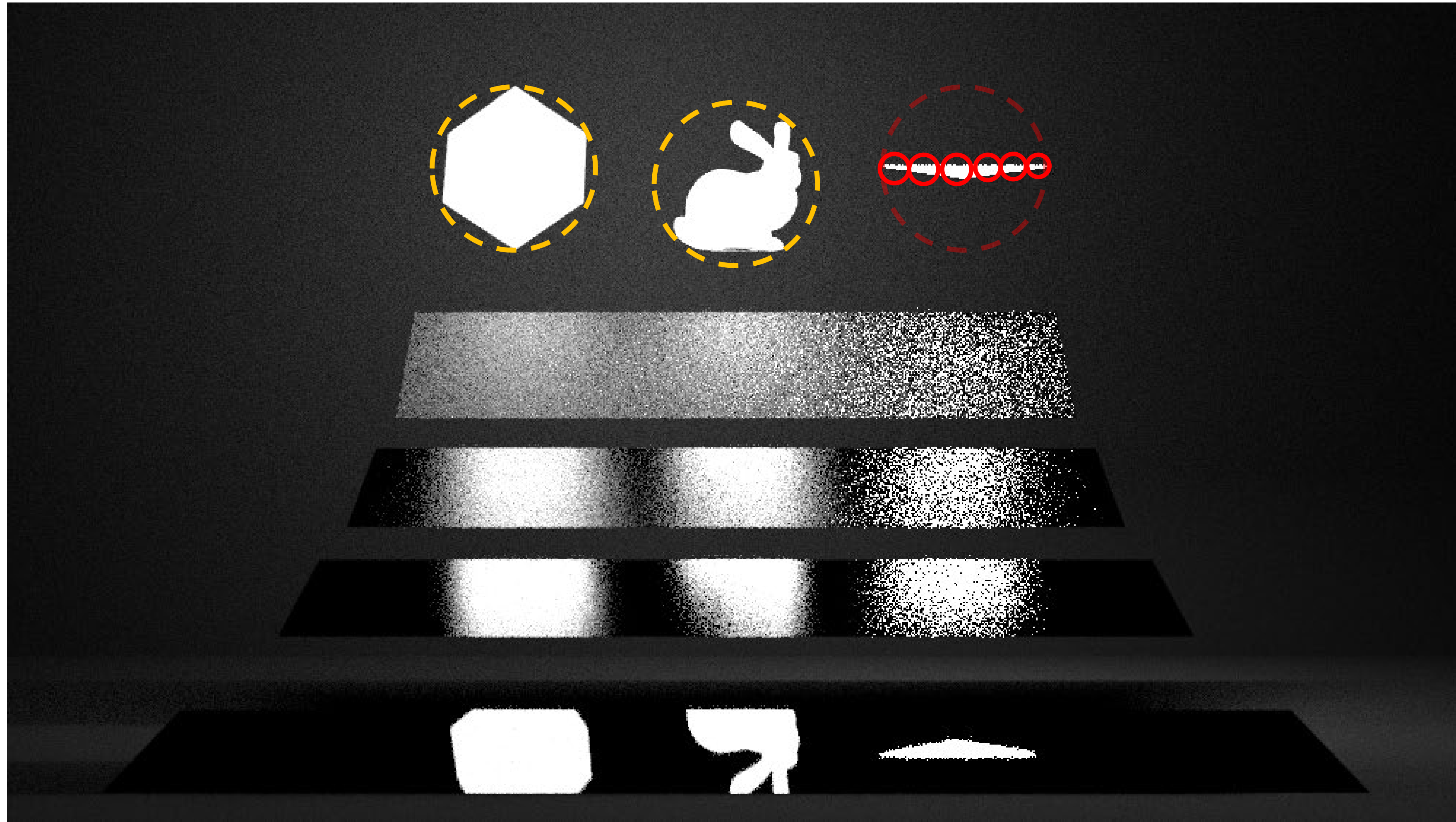
Extended to Any Shape Light



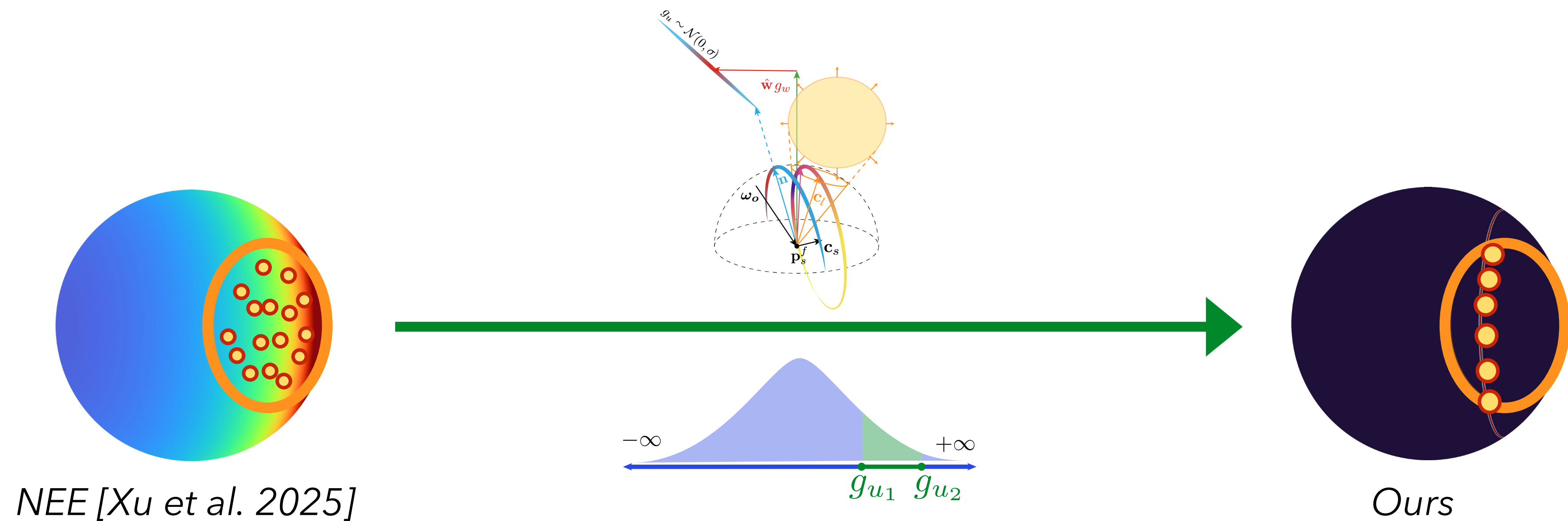
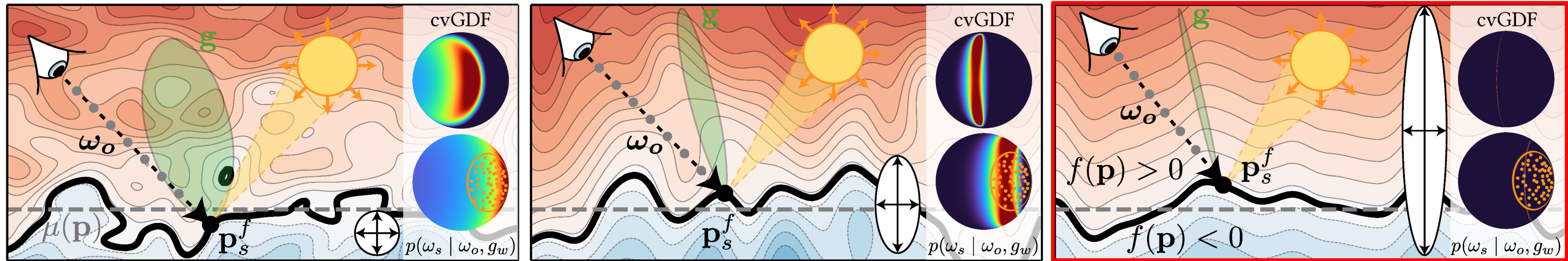
Limitation & Future Work



Limitation & Future Work



Conclusion



Conditional Product Next Event Estimation for Gaussian Process Implicit Surface

Thanks!



NSF Award 2440472



Kehan Xu



Wojciech Jarosz

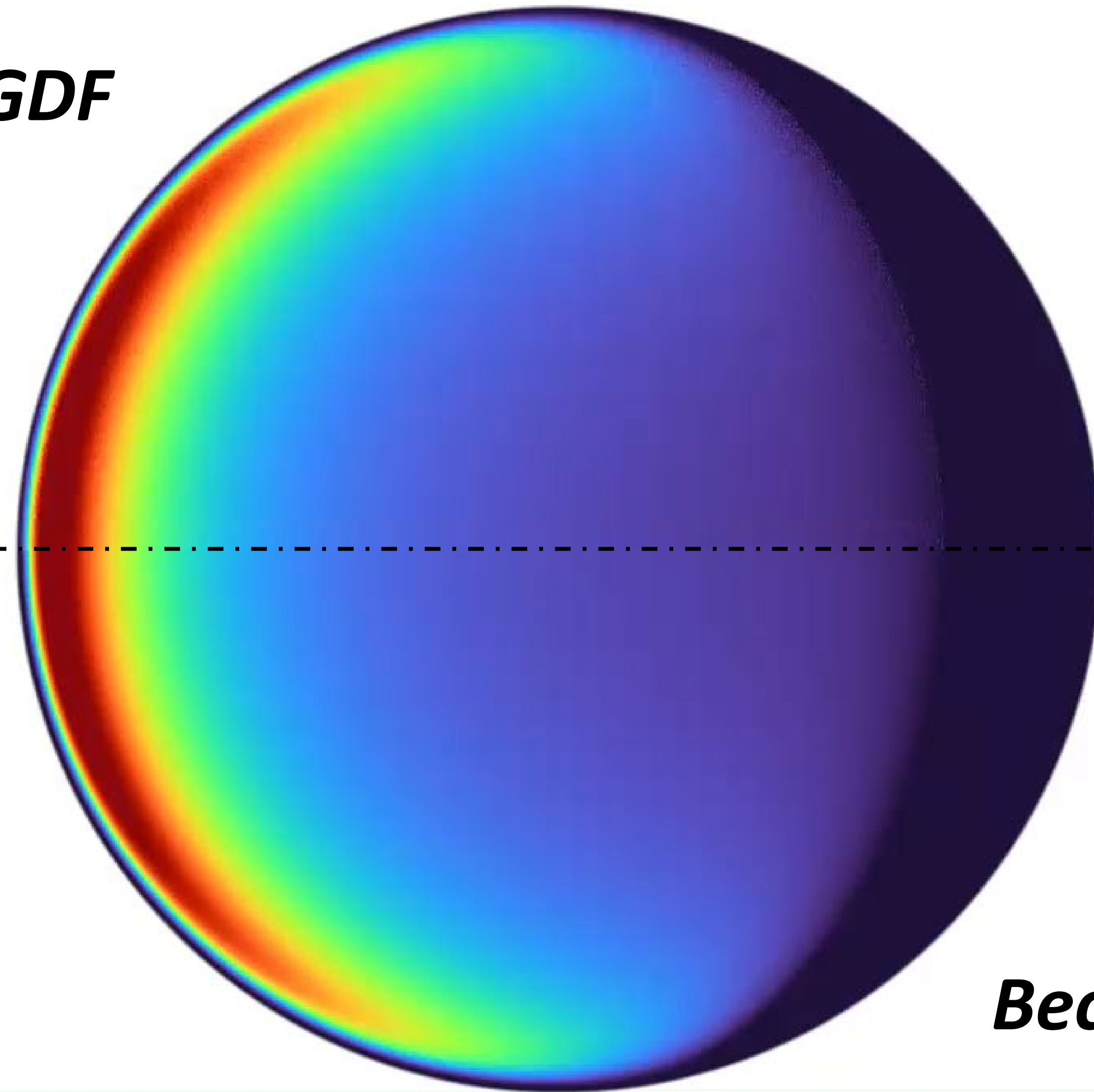
Dartmouth College





Heightfield GPIS with Beckmann Distribution

Heightfield GPIS cvGDF



Beckmann – Smith vNDF